7 - 11 July 2014

Kimberley - Northern Cape

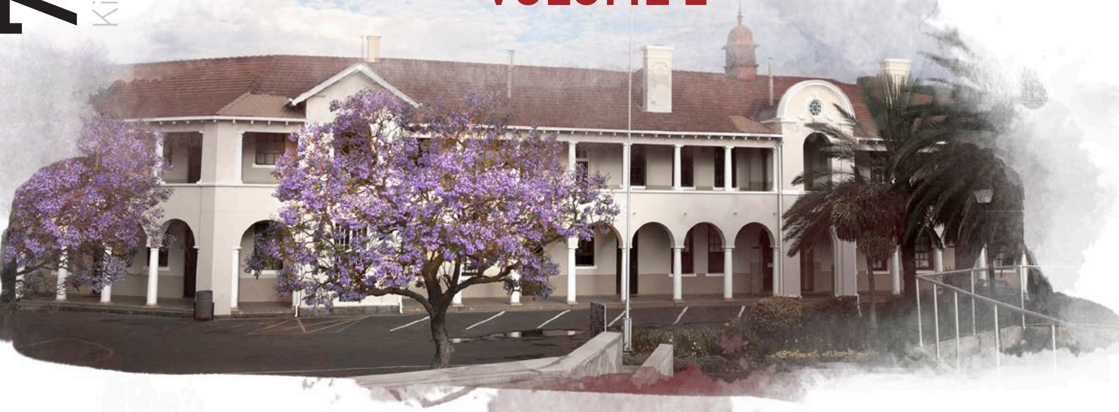
2014

AMESA

demystifying mathematics

Proceedings of the 20th Annual National Congress of the Association for Mathematics Education of South Africa

VOLUME 2



Editors:
Mandisa Lebitso
and Anne Maclean

Proceedings of the 20th Annual National Congress of the Association for Mathematics of South Africa

Volume 2

Demystifying Mathematics

07 - 11 July 2014

Diamantveld High School
Kimberley

Editors: Mandisa Lebitso and Anne Maclean

Association for Mathematics Education of South Africa (AMESA)

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FOREWORD

The theme of the 2014 AMESA Congress is: "Demystifying mathematics".

Why the need to demystify mathematics?

When was the last time you got up from your sofa to change the channel on your TV set or made a call from a public pay-phone or posted a letter to a friend? The world is changing at a rate as never before.

In the 21st century, scientific and technological innovations have become increasingly important as we face the benefits and challenges of both globalization and a knowledge-based economy. To succeed in this new information-based and highly technological society, students need to develop their capabilities in STEM to levels much beyond what was considered acceptable in the past.

Added to this, is the growing global recognition of the urgency of tackling a range of difficult and complex issues that impact on our human well-being. The world's population is estimated to rise from 7 billion to 8 billion by 2030 and this, against the backdrop of declining resources and changing climate conditions, means re-shaping where we live and how we live. Finding a way to adapt, to be efficient and sustainable, will require knowledge from many different sources.

In the South African context, where the number of learners taking mathematics and physical sciences at school level is declining, and the quality of matric results in these two gateway subjects is catastrophic, the pool from which the country is to grow its knowledge-based economy is less than sufficient.

South Africa has the third highest unemployment rate in the world for people between the ages of 15 to 24, according to the World Economic Forum (WEF) Global Risk 2014 report. The report estimates that more than 50% of young South Africans between 15 and 24 are unemployed. The quality of schooling, in particular, numeracy and mathematics competency, is closely linked to unemployment.

If we are to ensure a healthier economy for South Africa, better living conditions for all South Africans and a greener and more prosperous world for future generations, then the solutions have to be found in how we respond to 'demystifying' mathematics.

We hope that the presentations and deliberations at the 2014 congress, as well as the papers in the Proceedings, will go a long way towards addressing these pressing issues.

Mandisa Lebitso and Anne Maclean

July 2014

REVIEW PROCESS

The papers accepted for publication in this volume of the Proceedings (*Long Papers and short papers*) were subjected to triple-blind peer review by three experienced mathematics educators. The academic committee considered the reviews and made a final decision on the acceptance or rejection of each submission, as well as changing the status of submission. Authors of accepted submission were given the option of submitting an extended abstract rather than their full submission for publication in the publication elsewhere.

Number of submissions:	116
Number of plenary paper submissions:	5
Number of long paper submissions:	30
Number of short paper submissions:	11
Number of workshop submissions:	43
Number of '*How I teach*' paper submissions:	14
Number of poster submissions:	0
Number of submissions accepted:	103
Number of submissions rejected:	4
Number of submissions withdrawn by authors:	9

We thank the reviewers for giving their time and expertise to reviewing the submissions.

Reviewers:

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Avhasei Tsanwani
Anelize van Biljon
Lyn Webb

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2-Hour Workshops

1-Hour Workshops

How I Teach Papers

WHAT MATHEMATICS IS ON OFFER HERE?

Erlina Ronda and Jill Adler

Wits Maths Connect Project, University of the Witwatersrand

TARGET AUDIENCE: Senior phase, FET teachers and Teacher educators

DURATION: 2 hours

MAX. NO. OF PARTICIPANTS: 30

Studies show that the kind of examples we use, the way we set and sequence them, the way we name and talk about them together with what we accept or ignore as explanation and justification of the procedures we do with them and the nature of learners' participation during the lesson, greatly influence not only learners' understanding of the topic of the lesson but also the nature of mathematics that is communicated to them. It is the objective of the workshop to bring all these elements into focus so that teachers can consider them when planning and implementing their lessons. The workshop will feature a sample lesson on quadratic inequality. One of the presenters will teach the lesson with participants acting as learners. The participants and the workshop facilitators will then analyse the lesson using the analytic tool developed by Wits Maths Connect Project. The tool highlights the key elements of mathematics lessons the project recommends teachers should focus on in teaching.

MOTIVATION FOR WORKSHOP

Studies show that the kind of examples we use, the way we set and sequence them, the way we name and talk about them together with what we accept or ignore as explanation and justification of the procedures we do with them and the nature of learners' participation during the lesson, greatly influence not only learners' understanding of the topic of the lesson but also the nature of mathematics that is communicated to them. It is the objective of the workshop to bring all these elements into focus so that teachers can consider them when planning and implementing their lessons.

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CONTENT OF THE WORKSHOP

The workshop will feature a sample lesson on quadratic inequality. One of the presenters will teach the lesson with participants acting as learners. The lesson will then be analysed using the analytic tool developed by Wits Maths Connect Project. The tool highlights the key elements of mathematics lessons the project recommends teachers should focus on in teaching.

Proposed allocation of activities

Introduction of WMCS project and overview of the workshop	10 min
Q&A about the Analytic Tool	20 min
Sample teaching of quadratic inequality	30min
Reflection and Analysis of the lesson using the tool (in small group)	40 min
Big group discussion and wrap up	20 min

GUIDE QUESTIONS FOR GROUP DISCUSSION

1. What do you think was the main object of the lesson?
2. What examples were used in the lesson? What kind of tasks were the learners asked to do?
3. How would you describe the explanations made by the ‘teacher’ and the ‘learners’?
4. How would you describe the interaction in the ‘class’?
5. In what ways and how do the examples, explanations, and class interaction promote the object of learning?

MENTAL MATHS WITHOUT WORKSHEETS OR MINI-TESTS: ACTIVITIES TO GET YOUR CLASS INTO MATHS GEAR

Sue Fourie

The Grove Primary School

TARGET AUDIENCE: Foundation Phase

DURATION: 2 hours

MAXIMUM NO. OF PARTICIPANTS: 30

MOTIVATION FOR THE WORKSHOP:

Daily mental mathematics encouraged by the Curriculum and Assessment Policy Standards (CAPS) can be well used by Foundation Phase teachers to get children thinking, and put them into a positive and active space for mathematics learning. Unfortunately this new aspect of the curriculum may be ignored or may be implemented using worksheets with twenty or so calculations for learners to do independently. The purpose of mental maths is to get children talking, laughing and ready (in-gear) to work on more mathematics. This workshop will engage participants in a variety of games and activities which involve young children having fun while thinking mathematically.

DESCRIPTION OF CONTENT OF WORKSHOP:

A series of games using cards, dice and our own bodies for number work

The activities and worksheets to be used in the workshop (maximum 8 pages)

The presenter will bring along the widely available resources for these games (dice and cards). A hand out describing each activity will be provided for future reference for the participants.

An abstract describing the level, nature and content of the workshop (200 words)

Are you running short of engaging mental maths activities for your Foundation Phase learners? Come and play some games which involve the whole class in interactive play and mathematical thinking. There are no expensive resources needed. Your own bodies (including your heads to think, and voices to laugh), dice and packs of cards can be great resources for flexible and fun engagement with number. Sue Fourie will share some of her tried and tested activities which she uses with Grade 3 learners (many of which have application for Grades 1 and 2 as well).

RIDER STRATEGIES FOR SOLVING SCHOOL GEOMETRY PROBLEMS

Rajendran Govender

University of Western Cape

TARGET AUDIENCE:	SP& FET Band Mathematics Teachers
DURATION:	2 hour workshop
MAXIMUM NO. OF PARTICIPANTS:	30

MOTIVATION:

With geometry now being a compulsory topic examined across grades 10, 11 and 12 as per Curriculum and Assessment Policy Statement (CAPS), many teachers who have not taught senior school geometry nor had studied geometry beyond school level or even done geometry at high school level /university level are now faced with the task of teaching geometry to their learners. This has created a large amount of anxiety on the ground for both teachers and learners. This workshop aims to assist mathematics teachers to facilitate the teaching of problem solving within a geometry context through selecting and using a relevant rider strategy (or combination of rider strategies) as the solving of the geometry rider may necessitate. Particular emphasis will be placed on the writing of proofs inclusive of building arguments through using theorems and definitions as stipulated in the Department of Basic Education 2014 grade 12 examination guidelines for mathematics.

CONTENT

In this workshop, we will be focussing on the solving of geometry problems through using the following kinds of rider strategies in selected ways:

1. The congruency approach
2. Direct application of theorem(s)
3. The algebraic approach
4. Use of other branches of mathematics
5. Reductio ad absurdum (Proof by Contradiction)
6. Analytic approach

ABSRACT

The workshop will focus Euclidean Geometry specified in CAPS for grades 10-12 and is aimed at engaging SP &FET mathematics teachers in solving geometry riders associated with quadrilateral, triangle and circle geometries. In so doing mathematics teachers will be working in pairs on a set of riders that invokes one (or a combination) of the abovementioned strategies. Participants will discuss and share with the group the strategy/strategies that used to solve a given rider as well as challenges experienced. Emphasis will be placed on the writing of proof arguments that is substantiated by relevant reason(s) as suggested in 2014 grade 12 examination guidelines for mathematics.

USING AN EMPTY NUMBER LINE WITH FOUNDATION PHASE LEARNERS

Tania Halls and Nicky Roberts

The Grove Primary School

TARGET AUDIENCE:	Foundation Phase
DURATION:	2 hours
MAXIMUM NO. OF PARTICIPANTS:	30

MOTIVATION FOR THE WORKSHOP:

Using number lines for calculations is encouraged in the Curriculum and Assessment Policy Standards (CAPS) and was even assessed in the Annual National Assessment (2013). For many teachers this calculation approach is new, foreign and even threatening. We were not taught like this – why change? Why another method or strategy? In this workshop Tania will briefly share some of the benefits she has noticed from encouraging the use of an empty number line with her Grade 3 classes. Most of the session will be focused on teachers trying our different strategies on their own number lines and becoming comfortably themselves with this strategy. The workshop will start off with a focus on adding and subtracting on an empty number line, and (time permitting, and depending on the experience of the group) will shift to multiplying and dividing on empty number lines.

DESCRIPTION OF CONTENT OF WORKSHOP:

The workshop will open with some explanation of the empty number line and its benefits for recording mathematical thinking, as well as a mental tool for calculation. This will be followed by activities relating to adding and subtracting on the number-line where flexibility in approach (which depends on the numbers being used) will be encouraged. Attention will be drawn to the distinctions between using the number line with a "take-away" approach to subtraction, and using the number line with a "difference" approach to subtraction. Participants will be given time to work collectively and individually on using this technique. If time permits, then some discussion and similar activity will be conducted relating to multiplying and dividing on number lines.

The activities and worksheets to be used in the workshop (maximum 8 pages)

A power-point slide presentation will guide the introduction to the workshop and include examples of learner work using empty number lines. This will be followed by workshop task flow (the tasks given to participants to work on). A handout of the presentation will be provided as printed handouts to the participants.

An abstract describing the level, nature and content of the workshop (200 words)

Were your Foundation Phase learners able to correctly answer the ANA (2013) question which presented a number-line for them to show their calculation? Why does the CAPS encourage working on number lines? Is this just another fancy new method, you have to learn? Tania will share her experiences of shifting her teaching to encourage empty number line work with her Grade 3 classes. Come and spend time with her getting yourself comfortable, fluent and confident in using an empty number line for your own calculations. The focus will be on adding and subtracting, but if there is time and interest Tania can also share her experiences in using this for multiplying and dividing.

A FORMATIVE ASSESSMENT LESSON: MATCHING GRAPHS, STORIES AND TABLES

Marie Joubert, Barrie Barnard and Claire Blackman

AIMSSEC

TARGET AUDIENCE: Secondary phase
DURATION: 2 hours
MAXIMUM NUMBER OF PARTICIPANTS: 30

MOTIVATION:
This workshop covers the important idea of formative assessment, which is challenging for many teachers. It is based on a lesson on time and distance graphs, which includes extensive guidance for teachers, and which is written specifically as a 'formative assessment lesson'. Participants will learn about new ways to use formative assessment and will also be exposed to cutting-edge thinking about teaching the specific mathematics topic of time-distance graphs.

DESCRIPTION OF THE CONTENT OF THE WORKSHOP:

- Group discussion on formative assessment (15 minutes)
- Small group work on the task (30 minutes)
- Whole group work on the lesson as written (30 minutes)
- Whole group viewing of video of students working on the task (15 minutes)
- Small group discussion on formative assessment opportunities in the video (20 minutes)
- Wrap up (10 minutes)

Activities and worksheets to be used: Attached.

ABSTRACT:

This workshop aims to extend understandings of formative assessment. Participants are introduced to a 'Formative Assessment Lesson' produced by a team at the University of Nottingham, in the UK. The lesson is based around a task in which learners work in groups to match three sets of cards: one with distance-time graphs, one with stories and one with tables of data.
The workshop begins with a short discussion about current understandings of 'formative assessment'. Participants then take the role of learners to complete the task, all the time watching out for opportunities for formative assessment through their own discussions.

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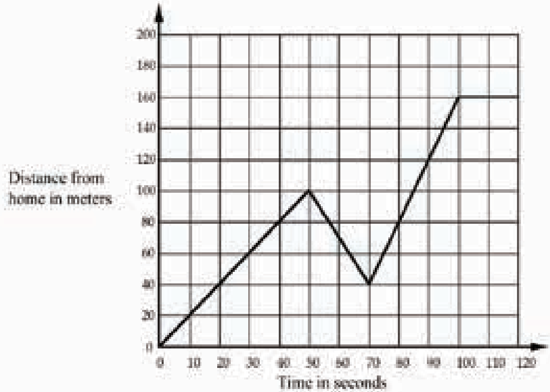
To continue, participants take the role of the teacher, as they are introduced to the full 'lesson', which includes: extensive guidance for the teacher; a diagnostic pre-lesson task for the students; the task itself; and a post-lesson task. (The full lesson can be found at http://map.mathshell.org/materials/download.php?fileid=1521). Key points relating to formative assessment are drawn out from the guidance.

Finally video of students (in the UK) working on the task will be shown. Participants will discuss how and when formative assessment is well used.

Journey to the Bus Stop

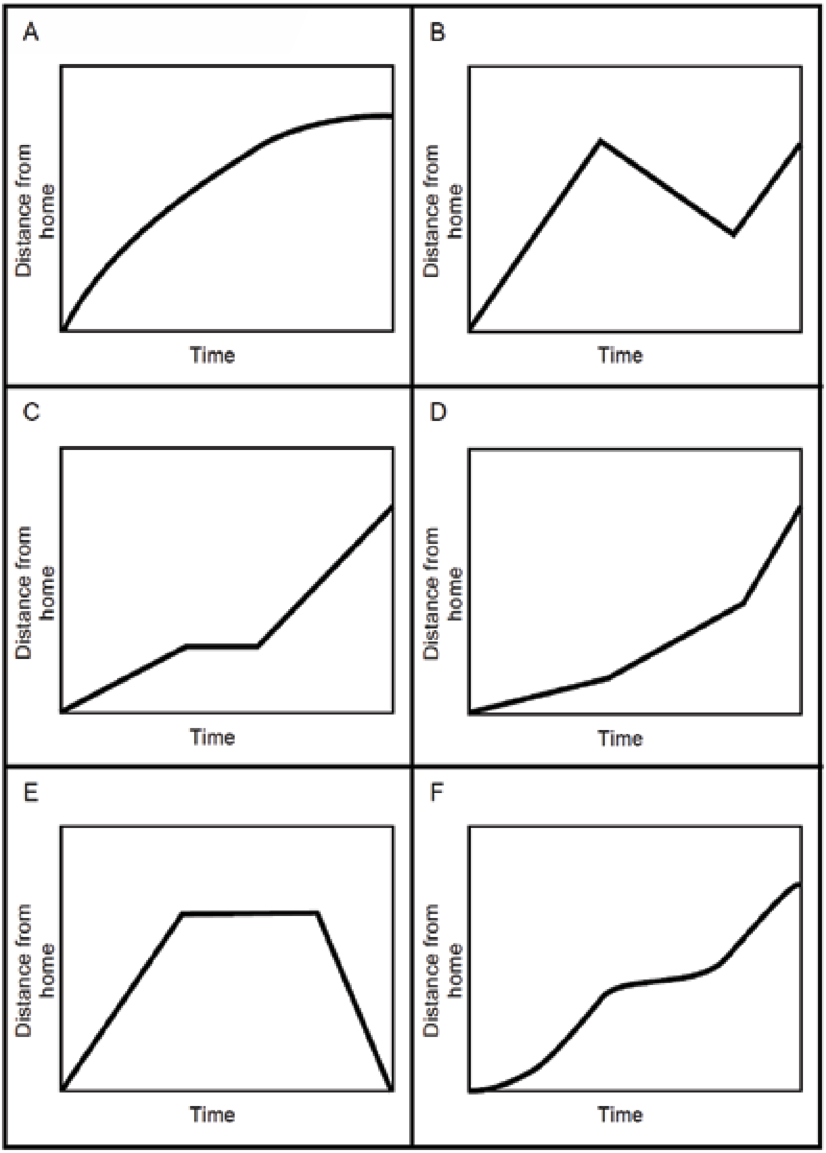
Every morning Tom walks along a straight road from his home to a bus stop, a distance of 160 meters. The graph shows his journey on one particular day.



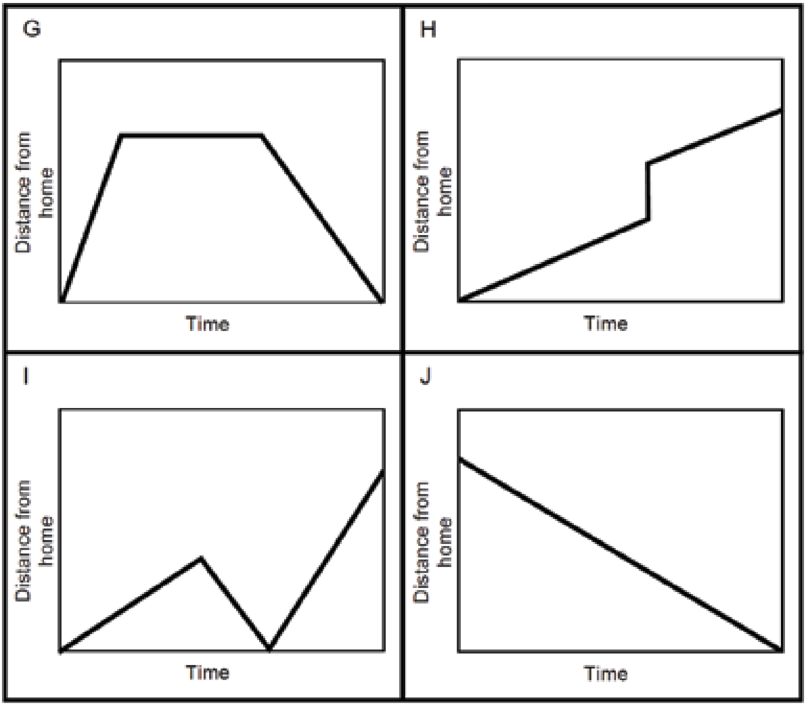
1. Describe what may have happened.
   You should include details like how fast he walked.

2. Are all sections of the graph realistic? Fully explain your answer.

Card Set A: Distance–Time Graphs



Card Set A: Distance–Time Graphs (continued)



Card Set B: Interpretations

1 Tom ran from his home to the bus stop and waited. He realized that he had missed the bus so he walked home.	**2** Opposite Tom's home is a hill. Tom climbed slowly up the hill, walked across the top, and then ran quickly down the other side.
3 Tom skateboarded from his house, gradually building up speed. He slowed down to avoid some rough ground, but then speeded up again.	**4** Tom walked slowly along the road, stopped to look at his watch, realized he was late, and then started running.
5 Tom left his home for a run, but he was unfit and gradually came to a stop!	**6** Tom walked to the store at the end of his street, bought a newspaper, and then ran all the way back.
7 Tom went out for a walk with some friends. He suddenly realized he had left his wallet behind. He ran home to get it and then had to run to catch up with the others.	**8** This graph is just plain wrong. How can Tom be in two places at once?
9 After the party, Tom walked slowly all the way home.	**10** Make up your own story!

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Card Set C: Tables of Data

P

Time	Distance
0	0
1	40
2	40
3	40
4	20
5	0

Q

Time	Distance
0	0
1	10
2	20
3	40
4	60
5	120

R

Time	Distance
0	0
1	18
2	36
3	54
3	84
5	120

S

Time	Distance
0	0
1	40
2	80
3	60
4	40
5	80

T

Time	Distance
0	0
1	20
2	40
3	40
4	40
5	0

U

Time	Distance
0	0
1	30
2	60
3	0
4	60
5	120

V

Time	Distance
0	0
1	20
2	40
3	40
4	80
5	120

W

Time	Distance
0	0
1	45
2	80
3	105
4	120
5	125

X

Time	Distance
0	120
1	96
2	72
3	48
4	24
5	0

Y

Make this one up!

Time	Distance
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

Z

Make this one up!

Time	Distance
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

USING THE CALCULATOR FOR FUNCTIONS IN THE FET BAND

Rencia Lourens
RADMASTE Centre, University of the Witwatersrand

TARGET AUDIENCE: FET Band Mathematics teachers.

DURATION: 2-hour workshop.

MAXIMUM NO. OF PARTICIPANTS: 30 participants

MOTIVATION

Functions form 35% of the Grade 12 paper 1, 45% in Grade 11 and 30% in Grade 10 (CAPS). The calculator can be used to support the calculations needed to draw and interpret the graphs of the functions.

CONTENT

The calculator will be used to support various aspects of the functions included in the FET CAPS.

20 minutes:	Intersection of two graphs
20 minutes:	Turning point and axes of symmetry of quadratic function
20 minutes:	Intercepts with axes of quadratic function
20 minutes:	Vertical asymptotes of reciprocal function
30 minutes:	Equations of the linear, quadratic and exponential dunction
10 minutes:	Wrap up

ABSTRACT

Functions and the graphs of functions form a substantial part of the FET work. Using a calculator can assist learners to understand the feautures of the functions. In this workshop we will focus on the linear, quadratic and reciprocal function.

WORKING WITH LEARNERS TO FIND THE EQUATION OF A STRAIGHT LINE FROM THE GRAPH

Julia Mabiletsa, Molatelo Racheky and Craig Pournara

Wits School of Education, University of Witwatersrand

The purpose of the workshop is to provide a platform for teachers from the Wits Maths Connect Secondary Project to share their professional development experiences with colleagues. The teacher-presenters have previously participated in professional development where they engaged with similar activities. In the proposed workshop teachers will present how they attempted the conceptual development of finding the equation of a straight line from the graph. The approach involved an alternative to the purely procedural approach to finding the linear equation. Workshop activities will focus on the development of the equation of the linear function and extending to its many representations.

TARGET AUDIENCE:	Grade 9-10 teachers wanting to build conceptual understanding of the straight line
DURATION:	2 hours
MAXIMUM PARTICIPANTS:	40

MOTIVATION FOR THE WORKSHOP

In our work in the Wits Maths Connect Secondary Project we have seen poor learner performance in a Grade 10 Learning Gains post-test. For example at the end of Grade 10, 65% of learners were able to write down the x-intercept and y-intercept of a straight line, given the graph. However, less than 4% could correctly determine the equation of the straight line.

There is much evidence from both internal and external school mathematics examinations that the majority of learners perform below expectations. The divide between what teachers teach and what learners do is ever widening. In the project we believe it is important to examine our teaching practices, and to pay more careful attention to the *opportunities to learn* that we offer our learners.

Given that more learners are able to write down the x- and y-intercepts in the post-test, it seems reasonable to assume that the poor performance in producing the equation stems in part from difficulties with determining the gradient, and from learners' poor conceptual grasp of the properties of a straight line.

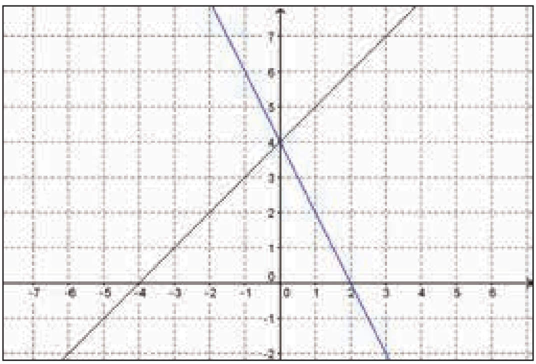
We believe that by addressing the concept of gradient, and by working with different representations of it, we might provide useful opportunities for our learners to learn. In the workshop participants will engage with the activities we have developed, and presenters will share experiences of working with learners on the activities.

Activity	Time (min)
Introduction & rationale	10
Activity 1: Finding the gradient and equation, given graph	25
Activity 2: Finding the gradient and equation, given a table	25
Activity 3: Finding the gradient using the gradient formula, then finding the equation	25
Connecting the activities together	15
Discussion and conclusions	20

An example of a workshop task is included below:

The diagram shows the graphs of 2 straight lines

a) What is the same about the lines?
b) What is different?
c) Will these graphs have the same equation?
    i) What will be the same in their equations?
    ii) What will be different?
    iii) What do we know about the equations?
    iv) What do we still need to find out?



HOW TO USE THE CASIO FXCG20 GRAPHIC CALCULATOR TO DRAW GRAPHS

Mirriam Mafojane

TARGET AUDIENCE: FET Band Mathematics teachers & Engineering

DURATION: 2-hour workshop

MAXIMUM NUMBER OF PARTICIPANTS: 30

MOTIVATION:

A practical approach to Mathematics using the FX-CG20 Calculator.

The Graphic application is for the graphical representation of functions and their investigation. It has two main windows: the Graphic editor to enter the function terms, and the Graphic window to show graphs of functions. In the Graphic editor, up to 20 terms can be entered (Y1-Y20)

Graphic calculators are complex and powerful tools for the modern teaching of mathematics. This brief guide is intended provide an introduction on the key applications and functionality of the Fx-CG20

CONTENT OF THE WORKSHOP:

1. *Graph*
2. *Graphics application,*
3. *graphical representation*
4. *functions, graphical analysis*

ABSTRACT:

Students can create a wide variety of graphs over real-life visual backgrounds. The use of real-life visuals makes it fun and easy to study various aspects of geometry, including the drawing of shapes, movement, and similarity relations. Students can search for and plot curves found in nature and their surroundings. Analysis of the plotted data deepens understanding of the function.

BASIC FUNCTIONS:

Graphing

- Rectangular coordinate graphing, Polar coordinate graphing
- Integration graph
- Parametric function graphing, Inequality graphing
- Trace, Zoom (box zoom, zoom in, zoom out, auto zoom)
- Table and Graph
- Dual Graph (table and graph, graph and graph)
- Sketch (tangent line, normal line, inverse function)
- Solve (root, minimum, maximum, intersection, integration)
- Dynamic graph
- Conic section graph
- Recursion graph
- Picture Plot (pre-installed software)

ACTIVITIES THAT ILLUMINATE AND VISIBILIZE FUNCTIONS AS OBJECTS

Shadrack Moalosi
University of the Witwatersrand, Wits School of Education, Wits Maths Connect the
Anancia Guveya
Marlboro High School, Johannesburg East District
Comfort Chigabo
Equinisweni Secondary School, Johannesburg East District

Purpose of the workshop is to afford teachers from the Wits Maths Connect professional development a space to share their PD experiences with colleagues. In the proposed workshop teachers will present to Grades (9 - 12) mathematics teachers some novel ways of teaching Functions. These are ways that aim to help learners perceive functions as constituting a relationship. The workshop will have three foci. First foci will engage participants in an activity that provides teachers with ways of seeing and explaining the horizontal shift in Functions. Second foci will engage participants with ways of helping learners move from perceiving functions as process to seeing them also as objects. The third foci will engage participants in an activity that illuminates and visibilizes the relationship between functions, domain and range. The workshop will cater for 30 teachers who will work in groups of 5 on selected activities. The duration of the workshop is 2-hours. At the beginning of the workshop we will use 10 minutes to introduce the workshop. Workshop activities will be allocated 1hr 20 minutes and the remaining 30 minutes will be used to reflect on the activities and for general discussions about the workshop.

MOTIVATION FOR THE WORKSHOP

This workshop is important for participants since it shares experiences/ideas and novel ways of explaining the Horizontal Shift in Functions. Functions are one of the topics in the mathematics curriculum which presents most challenges to learners. There is sufficient evidence that suggests that majority of our learners perform below national expectations in this area of mathematics (Adler, 2010; Simkins, 2013; Spaull, 2013). The 2012 report on the National Senior Certificate Examination (DoE, 2012) provides information that emphasizes the magnitude of the problem of low performance. We have selected to focus our workshop on Functions on account of its role as a gateway to University mathematics.

The essence of Functions in secondary school mathematics is a passage learners need to navigate in order to access further and harder university mathematics (Tall, 2010a). As such, there is greater need, through congresses such as AMESA, for teachers to share novel ways of teaching to minimize obstacles learners encounter along the passage to higher mathematics. We believe the workshop brings an alternative way of teaching Functions in ways that might minimize these obstacles, and that the workshop will provide teachers with opportunities to improve their teaching quality and increase learning opportunities for learners, particularly in the domain of functions. One of the obstacles we hope to address is the one where learners view graphs of Functions as pictures rather than relationships (Adler, 2010) .

In the Wits Maths Connect (WMC) project, we have noticed learners failing to obtain a solution from two intersecting graphs, preferring to solve equations of the graphs simultaneously which in our opinion was an unnecessary waste of time since we thought it could been much easier and quicker to obtain the solution from the intersection of two graphs. Now what was amazing was the fact that most learners, who had chosen the algebraic manipulation route, could not obtain the correct solution. We believe that learners could not obtain solutions from the graph because they perceived it as a picture. We associate learners' perception of graphs of Functions as pictures with teaching that always move from algebraic equation, to producing a table of values, to plotting and joining of points to produce a graph and where the graph is the product. We think this approach obscures the essence of Functions, (i.e. Functions as relationships between variables) from learners. In the workshop we propose to demonstrate how Functions may be taught in ways that might enable learners to see graphs of Functions as representing relationships that can be studied and to see them as having properties that can be analyzed to obtain solutions (Tall, 1997a). But most importantly we intend to demonstrate how all the four ways in which Functions may be represented are independent of each other and can be used to complement each other.

DESCRIPTION OF WORKSHOP CONTENT

The content of the workshop will be broken down as follows:

Introduction: (10 minutes)
The workshop will begin with an introduction of presenters and this will be followed by explanations about the rationale (for selecting Functions; specific content on Functions; our approach; and target group), purpose, and the design of workshop activities.

Workshop Presentation: (1hr 20 min)
Target Audience: Our main target is teachers offering mathematics at Grades (9 -12)
Duration: The workshop is designed to fit within a 2-hour Slot
Maximum no. of participants: Thirty (30)

Activity 1: Investigating the horizontal shift using a piecewise function (30 minutes)
In this activity participants will be given non-standard or piecewise functions and translation rules to observe and analyze the horizontal shift on the given graphs. The benefit of this activity is that it provides the explanatory framework which teachers can use to explain how and why the graph of g(x) might when transformed to g(x-1) and g(2x). **Figures 1, 2, 3** and **Table 1** are examples of resources to be used in Activity 1.

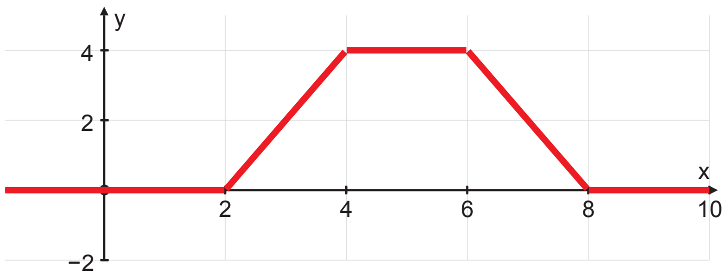


Figure 1: Graphical representation of the piecewise Function

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x	0	1	2	3	4	5	6	7	8	9	10
$g(x)$											
$g(x-1)$											
$g(2x)$											

Table 1: Numerical representation of the piecewise Function

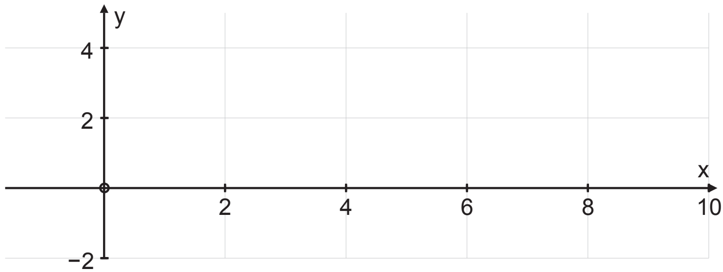

Figure 2: Cartesian plane for the graph of g(x-1)

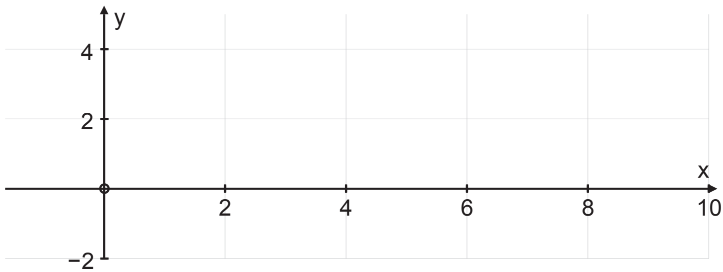

Figure 3: Cartesian plane for the graph of g(2x)

Activity 2: Moving from functions as process to functions as objects (30 minutes)
In this activity participants, working in groups of 5, will be issued with worksheets containing four conditions. Participants will them be required to find 8 functions that satisfy the given conditions. The activity requires participants to know and work with properties of functions simultaneously; hence when used in the class it will facilitate learners' global view of functions. Thus, a move from functions as process to functions as objects. **Figure 4** is an example of a resource for Activity 2.

Fill the spaces in the three intersecting circles with functions that satisfy these three conditions

A: The range includes the value $y = -1$
B: The graph is of the form $y = f(x-3) + a$
C: The graph passes through the fourth quadrant

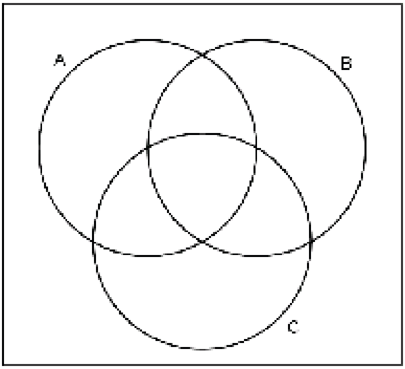

Figure 4: Intersecting circles

Activity 3: Matching functions with graphs, domain and range (30 minutes)

Working in groups of 5, participants will be provided with 36 cards containing function, graph, domain and range. Each group will sort the cards into piles of four such that each pile contains a function and its matching graph, domain and range.

SUMMARY AND CONCLUSION

Activity: Participants' reflections on activities and discussions: **(20 minutes)**

In this part of the workshop, participants will reflect on the 3 activities, discuss and share experiences drawn from engaging with workshop activities.

ACKNOWLEDGEMENTS

This work is based upon the research and development of the Wits Mathematics Connect Project at the University of the Witwatersrand, supported by the FirstRand Foundation Mathematics Education Chairs Initiative of the FirstRand Foundation the Department of Science and Technology and the National Research Foundation (NRF). The workshop also drew its material from the TAM project in the UK. Any opinion, findings and conclusions or recommendations expressed in this material are those of the author and do not necessarily reflect the views of the FRF, DST or NRF.

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PROBABILITY

Oetsi Mokhobo

STATISTICS SOUTH AFRICA

DURATION: **2 HOURS**
NUMBER OF PARTICIPANTS: **30**
TARGET AUDIENCE: **FET (GRADE 10 – 12)**

MOTIVATION FOR WORKSHOP

Probability is a concept that prior to the curriculum review, formed part of content that was examined in an optional Paper 3. The implementation of Curriculum and Assessment Policy Statement (CAPS) has made the teaching of probability compulsory. As a result, all mathematics teachers are faced with a challenge of having to handle probability, a concept that was either not included in their formal professional training or have limited exposure or experience toward teaching it to learners. The purpose of conducting a workshop on probability is an attempt to provide an opportunity for teachers to interact with probability concepts and content that are necessary for teaching and learning.

CONTENT OF THE WORKSHOP

Few interesting and relevant data from Census 2011 will be shared with the audience. This worksheet consists of ten activities including the following:

- Calculations of theoretical and relative frequency of events happening;
- Draw and interpret Venn diagrams;
- Use Venn diagrams to determine the probability of events happening;
- Defining mutually exclusive events and
- Using probability rules to determine probabilities.

Proposed time allocation for activities

1. Interpretations / discussion of Census 2011 findings	20 Min
2. Probability scale and concepts	20 Min
3. Probability problems from worksheets (hand outs)	60 Min
4. Summary of key concepts and handing out of study guides	20 Min

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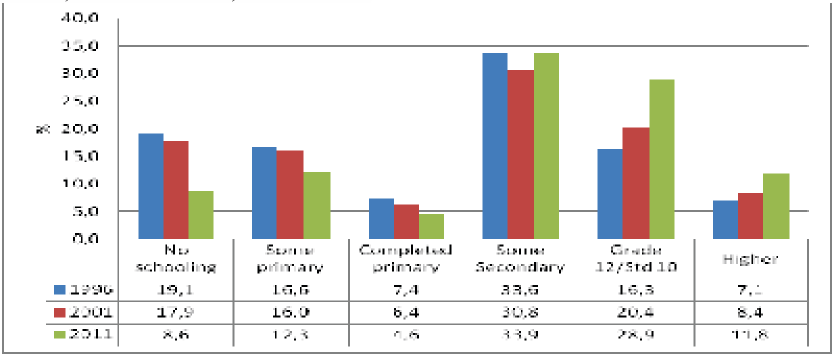
<u>Activity1. (Census 2011)</u>

Highest level of education attained amongst persons aged 20 years and older and above, Censuses 1996, 2001 and 2011



	No schooling	Some primary	Completed primary	Some Secondary	Grade 12/Std 10	Higher
1996	19,1	16,6	7,4	33,6	16,3	7,1
2001	17,9	16.0	6,4	30,8	20,4	8,4
2011	8,6	12,3	4,6	33,9	28,9	11,8

1) What can you deduce from the P(No schooling) of 2011 and 1996?
2) The percentage of people, who completed primary education decreased in 2011 than in 1996. Why?
3) The percentage of people, who completed grade 10 and higher institution of learning increased. Why?

CALCULATING PROBABILITY

Theoretical probability

ACTIVITY.2

Give each of the answers in (b)

i) as a common fraction in simplest form,
ii) as a decimal fraction (correct to 2 decimal places)
iii) as a percentage (correct to 1 decimal place).

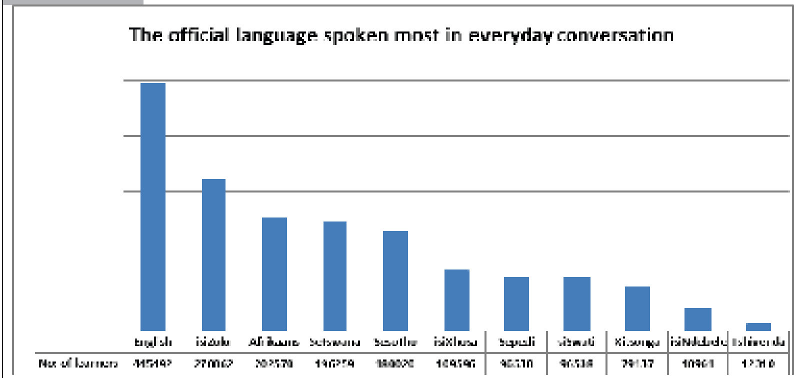
A fair die is rolled once.

a) List the elements of the sample space.
b) What is the probability that you will get
   i) A six?
   ii) An odd number?
   iii) A seven?
   iv) More than 2?
   v) Less than 10?

Relative frequency

ACTIVITY.3

The bar graph below is taken from 2009 Census@School. A sample of all the learners in South Africa was asked which of the official languages they spoke most in everyday conversation. (*The language used in everyday conversation is the language you use most of the time when talking and listening to others*). The bar graph shows their answers.



a) How many learners were surveyed?

b) Estimate the probability (as a percentage correct to 1 decimal place) that a learner selected at random from the sample

i) Speaks mainly English in everyday conversation

ii) Speaks mainly isiZulu OR Afrikaans in everyday conversation

c) In Census 2011 it was found that in South Africa, with a population of 51 770 560. 9, 6% speak English and 36, 2% speak isiZulu or Afrikaans in everyday conversation. Which results, 2009 Census@School or Census 2011, give better estimates? Give reasons for your answer.

Drawing Venn diagrams

ACTIVITY.4

A sample space S consists of whole numbers from 20 to 29 inclusive.
Event A consists of the multiples of 4 in S.
Event B consists of the factors of 420 in S.
Event C consists of the multiples of 5 in S.
Event D consists of the multiples of 3 in S.

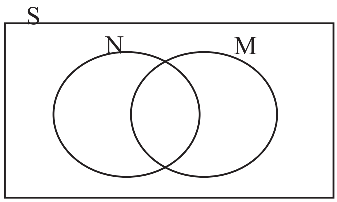
a) List the elements in
   i) S   ii) A   iii) B   iv) C   v) D
b) Draw Venn diagrams to show
   i) Sample space S and event A
   ii) Sample space S, event A and event B
   iii) Sample space S, event C and event D

Interpreting Venn diagrams

ACTIVITY.5

Draw six Venn diagrams like the one given.
On each one shade one of the following:

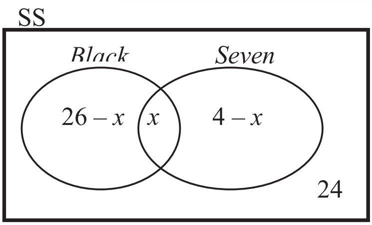
1) N
2) N *and* M
3) N *or* M
4) N but not M
5) M but not N
6) Neither M nor N



Venn diagrams showing the number of outcomes in events

ACTIVITY.6

The Venn diagram illustrates the number of playing cards in a pack of playing cards which are black as well as the number of cards that are sevens. Use the Venn diagram to answer the following:



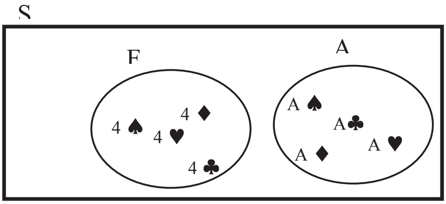
a) How many cards are there in a pack of cards?
b) How many black cards are there in a pack of playing cards?
c) How many sevens are there in a pack of playing cards?
d) How many cards are black *or* seven?
e) Find the value of x; where x is the number of black sevens in a pack of playing cards.
f) Check your answers by substituting for x and adding.

Finding a relationship between the number of outcomes in different events

ACTIVITY.7

The Venn diagram illustrates the number of playing cards in a pack of playing cards that are fours as well as the number of cards that are aces.

Let F be the set of fours in a pack of playing cards
Let A be the set of aces in a pack of playing cards



1) Use the Venn diagram to find the following:
   a) $n(F)$
   b) $n(A)$
   c) $n(F \text{ and } A)$
   d) $n(F \text{ or } A)$
   e) $n(F)+n(A)-n(F \text{ and } A)$

2) Is $n(F)+n(A)-n(F \text{ and } A)=n(F \text{ or } A)$?

Venn Diagrams showing the probability of events happening

ACTIVITY.8

Two events A and B have the following probabilities:

$P(A)=0{,}2;\ P(B)=0{,}4$ and $P(A \text{ and } B)=0{,}08$

Draw a Venn diagram to illustrate the situation

a) Determine $P(A \text{ or } B)$

b) Is $P(A \text{ or } B)=P(A)+P(B)-P(A \text{ and } B)$?

PROBABILITY RULES

ACTIVITY.9

In the 2009 Census@School, the Grade 10 to 12 learners were asked what type of home they stay in most of the time. 58, 6% of the learners answered that they live in a house (H) and 14, 8% live in a traditional dwelling (T).

a) Are ‘living in a house (H)’ and ‘living in a traditional dwelling (T)’ mutually exclusive?
b) Calculate the percentage of the Grade 10 to 12 learners who do not live in a house or traditional dwelling.
c) Draw a Venn diagram to show the percentage of learners who live in a house (H), the percentage of learners who live in a traditional dwelling (T) and the percentage of learners who do not live in a house or in a traditional dwelling.
d) Suppose one of the Grade 10 to 12 learners is selected at random. Determine
   i) *P(H)*
   ii) *P(T)*
   iii) *P(not H)*
   iv) *P(not T)*
   v) *P(H or T)*

ACTIVITY.10

In a the 2009 Census@School survey of 15 to 19 year olds, learners were asked what sport they would like to take part in. Below is data adapted from the database:

Age 15 -19 males and females sport	**%**
Athletics (A)	x
Volleyball (V)	21 %
Athletics and Volleyball	12 %
Neither of these sports	65 %

a) Draw a Venn diagram to illustrate the data given in the table.
b) Use the Venn diagram to determine the value of x.
c) Calculate the probability that a learner chosen at random:
   i) likes Athletics but not Volleyball
   ii) likes Athletics or Volleyball

REFERENCES:

- Stats SA, 2011, Data handling & Probability study guide: Grades, 10, 11and 12 HCD (Human Capacity Development.)
- Statistical release : P0301.4 Census 2011

USING GAMES TO CONSOLIDATE CALCULUS RULES AND CONCEPTS

Ingrid Mostert

Kelello Consulting

TARGET AUDIENCE: FET Phase

DURATION: 2 hours

MAXIMUM NO. OF PARTICIPANTS: 30

One of the challenges of teaching is finding the balance between developing understanding and practising skills. Often it is easier to make concept development interesting through visual representations or activities while practicing skills remains a tedious exercise. Using well designed mathematical games is one way of making the practice of skills less tedious and can, in some cases, be used to develop understanding.

In this workshop we will look at four games that can be used to teach calculus. The games address the need to convert expressions to an appropriate form before apply derivative rules; the difference between various rules (chain rule, product rule, quotient rule); the question whether two functions with the same derivative are necessarily the same; and the need to practice deriving functions.

MOTIVATION FOR RUNNING THE WORKSHOP

Participants will find this workshop useful because it will provide them with a means of engaging their learners in practicing some of the skills needed to comfortably differentiate functions. It will also provide them with a generic game that can be adapted to practice various skills.

DESCRIPTION OF CONTENT OF WORKSHOP

A brief introduction (10min) will look at the difference between mathematical games that are useful and those that aren't.

The participants will be divided into small groups and each of the games will be played in turn. The first three games will be play for approximately 15 minutes each after which participants will have a further 10 minutes to discuss the concepts that the game is trying to address.

The last game will be allocated the remaining 35minutes as it can be adapted to practice various skills. After playing the game, participants will have the opportunity to discuss how they would adapt the game to allow their learners to practice a particular skill.

At the end of the workshop, participants will be provided with the worksheets necessary for playing the four games in their own classes.

ACTIVITIES AND WORKSHEETS TO BE USED

The games have been attached to the email. I am aware that the guidelines state that not more than two pages should be taken from the same source but I feel that the four games which are all shared on http://busynessgirl.com/games/calculus/ are an excellent resource that teachers can use effectively in their classes.

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MODELS AND EXPLANATIONS FOR EARLY ADDITION AND SUBTRACTION

Quinton Nam and Hamsa Venkat

PHASE: Foundation Phase
TIME: 2 hours

The Wits Maths Connect Primary Project (WMCPP) has spent the past 3 years working with teachers in ten partner schools on improving mathematics teaching and learning and assessing the effectiveness of interventions developed by the (WMCPP).

Conversations with teachers and interviews and assessments with learners point to areas within early addition and subtraction that continue to be seen as problematic. This evidence shows that learners are often able to perform basic addition and subtraction calculations such as:

$9 + 7 = \square$ and $9 - 7 = \square$

Yet, they often have difficulty in solving missing subtrahend ($9 - \square = 2$), missing addend ($9 + \square = 16$) and missing start ($\square - 7 = 2$; $16 = \square + 7$) problems.

When we ask teachers how they deal with and explain these latter problem types, they have usually told us that they tell children to use rules like: ‘For $9 - \square = 2$, you just do $9 – 2$ and for $9 + \square = 16$, you do $16 – 9$’.

These rules allow children to produce the correct answer for the current problem, but they frequently remain unable to remember the rules well enough to solve similar problems independently.

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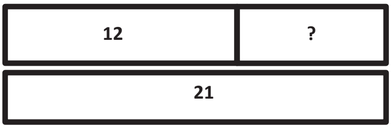
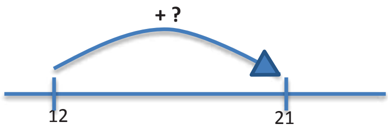
In order to aid teachers in dealing with the difficulties that learners experience when attempting such problems, this workshop aims to introduce and discuss two key models and associated explanations for early addition and subtraction problems: the part-part-whole bar model and the number line model. Our experience suggests to these models being useful for helping children to see the structure of problems and to use this structure to select appropriate problem-solving strategies. We will use these models in the context of both number sentences and word problems relating to early addition and subtraction. The workshop will incorporate opportunities for teachers to become familiar with the use of these models and accompanying explanations that support children's take up and use of these models in their own work.

e.g. 12 + [ ] = 21

Part-part-whole bar model:



Number line model



CHANGING THE WAY WE TEACH FUNCTION TRANSFORMATIONS WITH GEOGEBRA

Cerenus Pfeiffer

Stellenbosch University Centre for Pedagogy (SUNCEP)

TARGET AUDIENCE: FET teachers (Grade 10 – 12).

DURATION: 2 hours

MAXIMUM NO. OF PARTICIPANTS: Number of participants will depend on number of computers in the computer lab. Each participant must have access to a computer for this session.

MOTIVATION FOR WORKSHOP (ABOUT 2 OR 3 LINES)

- Many learners asked teachers: "Why do we do the opposite when a change is made inside the parenthesis. Why does $f(x-2)$ move the graph of f two units to the right? Teachers have often tried to explain away this uneasiness by telling students to 'do the opposite operation when the change is inside the parenthesis.' This approach, of course, leaves much to be desired, especially when the goal of effective mathematics instruction is to help students develop reasoning about why things happen." (Faulkenberry & Faulkenberry, Transforming the way we teach function transformations, 2010.)
- Workshop will focus on approaches that will try to avoid giving learners only procedures to apply and memorization of the transformation rules. Workshop will also focus on visualizing the function transformations with *GeoGebra*.
- Teachers will be exposed to how to integrate *GeoGebra* (dynamic software) in the teaching of function transformations.
- To expose teachers to mathematics material that facilitates self-exploration and self-activity with *GeoGebra*.

CONTENT OF WORKSHOP (ABOUT 5-10 LINES TO ENTICE THE AUDIENCE TO YOUR WORKSHOP)

Participants will:

- Explore *GeoGebra* to familiarise themselves with the working of the software.
- Create activities to investigate the types of transformation when given $-f(x)$, $f(-x)$, $f(x)-\ 2$, $f(x+3)$, $3f(x)$, $f^{-1}(x)$, etc.
- Draw explore the different transformations in the following functions:

- $y = a(x-p)^2 + q$
- $y = a(b)^{x+p} + q$
- $y = \frac{a}{x+p} + q$
- $y = a\sin(bx+p) + q$

PROPOSED TIME ALLOCATION FOR WORKSHOP ACTIVITIES

Short presentation on the need for learners to be active involve in the process of learning and the history of *Geogebra*.	**10 minutes**
Participants to familiarise themselves with the icons in *GeoGebra* that they will use during this session.	**15 minutes**
Creating of activities in *Geogebra* to investigate the types of transformation when given $-f(x)$, $f(-x)$, $f(x) - 2$, $f(x+3)$, $3f(x)$, etc.	**1h15minutes**
Related discussions and installation of software.	**20 minutes**

Function transformations with *GeoGebra*

1. $f(x) = 3(x-1)^2 - 8$. Type in Geogebra in Input bar 3(x-1)^2 – 8 and then Enter.

Turning point coordinates :

x –intercepts: Type in Geogebra in Input bar: **root(f)** and then press enter on keyboard.

Type in the following into the input bar and complete the table.

	Equation of transformed graph	Type of transformation	Coordinates of TP	x-intercepts	y-intercept
(a) $g(x) = -f(x)$					
(b) $h(x) = f(-x)$					
(c) $p(x) = f^{-1}(x)$					
(d) $q(x) = f(x) - 2$					
(e) $r(x) = f(x+3)$					
(f) $s(x) = f(x-1) + 3$					
(g) $t(x) = 3f(x)$					

2. $f(x) = x^2 - 2x - 8$

Type in the **Inputbar:** x^2 – 2x – 8 and press then enter on the keyboard.

For turning point: Type in inputbar : **extremum(f)** and then press enter on keyboard.

For x-intercepts: Type in Geogebra in Input bar: **root(f)** and then press enter on keyboard.

Type in the following into the input bar and complete the table.

	Equation of transformed graph	Type of transformation	Coordinates of TP	x-intercepts	y-intercept
(a) $g(x) = -f(x)$					
(b) $h(x) = f(-x)$					
(c) $p(x) = f^{-1}(x)$					
(d) $q(x) = f(x) - 2$					
(e) $r(x) = f(x + 3)$					
(f) $s(x) = f(x - 1) + 3$					
(g) $t(x) = 3f(x)$					

3. $f(x) = \frac{4}{x+2} - 3$.

Type in the input bar f(x) = 4/(x+2) – 3 and press then enter on the keyboard.

Equation of horizontal asymptote (HA):

Equation of vertical asymptote (VA):

Type in the following into the input bar and complete the table.

	Equation transformed graph	Type of transformation	Equation of HA	Equation of VA
(a) $g(x) = -f(x)$				
(b) $h(x) = f(-x)$				
(c) $p(x) = f^{-1}(x)$				
(d) $q(x) = f(x) - 2$				
(e) $r(x) = f(x + 3)$				
(f) $s(x) = f(x - 1) + 3$				
(g) $t(x) = 3f(x)$				

4. $f(x) = 2^{x+1} - 3$.

Type in the input bar f(x) = 2^(x+1) – 3 and press then enter on the keyboard.

Equation of horizontal asymptote:

Type in the following into the input bar and complete the table.

	Equation transformed graph	Type of transformation	Equation of HA
(a) $g(x) = -f(x)$			
(b) $h(x) = f(-x)$			
(c) $p(x) = f^{-1}(x)$			
(d) $q(x) = f(x) - 2$			
(e) $r(x) = f(x + 3)$			
(f) $s(x) = f(x - 1) + 3$			
(g) $t(x) = 3f(x)$			

5. $f(x) = sin(x + 30^o)$.

Type in the input bar g(x) = sin(x°+30°)

Maximum value :

Minimum value :

Period:

Amplitude:

Type in the following into the input bar and complete the table.

	Equation transformed graph	Amplitude	Period	Maximum value	Minimum value
(a) $-f(x)$					
(b) $f(-x)$					
(c) $f(x - 30°)$					
(e) $f(x + 60^{\circ}) - 3$					
(f) $f(2x)$					
(g) $f(\frac{x}{3})$					
a) $3f(x)$					

SO YOU WANT ACCURATE GRAPHS IN YOUR MATHEMATICS PAPER? USE GRAPH.EXE SOFTWARE FOR FREE!

Raadt P.K

Emang Mmogo Comprehensive School, Kimberley

TARGET AUDIENCE: FET – Educators of Mathematics.

DURATION: 2 hours

NO. OF PARTICIPANTS: Size of computer laboratory

MOTIVATION FOR RUNNING THE WORKSHOP

As a Provincial Examiner and SBA moderator, I have observed that despite the work done by government to reform the education system across the country and improve standards in schools, the 2013 SBA moderation reports still recommends that workshops be conducted for educators on how to use technology in setting credible Mathematics question papers.

Many educators in high schools are still deeply challenged in the area of producing error free Mathematics Assessment Tasks, particularly with regard to drawing mathematical diagrams and graphs. This is largely due to low levels of the knowledge and usage of mathematical software programmes. The consequences of this low level of mathematical software education may be long term as well as affect and limit the implementation of the new CAPS.

I also observed that over dependence on past years national question papers have become an unhealthy trend. This dependence stifles creativity and originality that is expected from teachers. A lack of competence in the usage of technology related skills stifled creativity in the construction of original assessment tasks. Some teachers lacking the capacity to develop their own assessment tasks and tests also feel hugely humiliated when they receive negative comments by Provincial and National SBA moderators.

USEFULNESS OF THE WORKSHOP

The workshop will expose educators not only to drawing these functions and graphs but to accurately label, copy and paste into Microsoft word for a professional final product. The workshop is designed for current FET Mathematics teachers who desire to develop skills that would allow them to produce error free question papers by making use of Nationally Accepted Mathematical Software Programme, Graph.exe.

Furthermore the results of this workshop will ensure better assessment tasks and assessment tools.

DESCRIPTION OF THE ACTIVITY

Teacher will download their own Graph.exe software to take home. They will be provided with worksheet material to use during the workshop. Worksheets developed and used during the workshops offers a major and invaluable resource for teachers' self-development in particular as well as hugely benefit the education system in general.

HOW WILL THE TIME BE BROKEN UP?

Summary of time and content	
Length	Topics
20 minutes	Introduction. Downloading the software.
40 Minutes	Drawing trigonometric graphs. Editing on Microsoft word.
30 minutes	Drawing a parabola, straight line and exponential graphs.
30 minutes	Drawing hyperbolic and cubic functions.

USING MICROSOFT EXCEL TO DRAW ACCURATE OGIVES, HISTOGRAMS AND FREQUENCY POLYGONS

Raadt P.K

Emang Mmogo Comprehensive School, Kimberley

TARGET AUDIENCE: FET – Educators of Mathematics and Mathematical Literacy.

DURATION: 2 hours

NO. OF PARTICIPANTS: Size of computer laboratory

HOW WILL THE TIME BE BROKEN UP?

Summary of time and content	
Length	Topics
50 minutes	Entering data on a spreadsheet Drawing an ogive.
40 minutes	Drawing a histogram.
30 minutes	Drawing a frequency polygon. Discussion .

MOTIVATION FOR RUNNING THE WORKSHOP

The aim of this workshop is to assist the educators who do not have the Autograph software to use the readily available software (Microsoft excel) to draw the statistical curves and graphs. It is a known fact that many educators use 'cut and paste' option when they finalise their test and examination papers and in the process producing a product that is below standard for learners and moderators.

The workshop will expose educators not only to drawing these curves and graphs but to accurately annotate copy and paste into Microsoft word for a professional final product.

DESCRIPTION OF THE ACTIVITY

Educators will be presented with grouped continuous data to enter into the spreadsheet cells and a step by step guide (worksheet) to use during the workshop and to take home. Throughout the workshop (all 3 sessions) educators will work with the same data items so as to immediately note the differences between ogive, frequency polygon and histogram.

USEFULNESS OF THE WORKSHOP

Hopefully the workshop will motivate the educators to use computer technology in teaching mathematics and in the administration in general.

Using Microsoft Excel to accurately draw ogive, histogram and frequency polygon

Activity 1:

1. **Opening Microsoft excel**
   a. Click on start
   b. Go to programmes (all programs on windows *XP*)
   c. Go to Microsoft office
   d. Then click on Microsoft excel 2007 (a table consisting of cells will appear)

2. **Entering data on cells**

The following table shows the ages of people on a Transnet commuter train one Saturday morning from Kimberley to Bloemfontein.

Age (in years)	No. of people
$10 < A \leq 20$	37
$20 < A \leq 30$	59
$30 < A \leq 40$	156
$40 < A \leq 50$	172
$50 < A \leq 60$	143
$60 < A \leq 70$	75
$70 < A \leq 80$	22
$80 < A \leq 90$	10

a. In order to draw the three graphs, we need to enter the following grouped continuous data in to cells

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Age (in years)	Mid-point	Upper limit	frequency	Cumulative frequency
$0 < A \leq 10$	10	5	0	0
$10 < A \leq 20$	15	20	37	37
$20 < A \leq 30$	25	30	59	96
$30 < A \leq 40$	35	40	156	252
$40 < A \leq 50$	45	50	172	424
$50 < A \leq 60$	55	60	143	567
$60 < A \leq 70$	65	70	75	642
$70 < A \leq 80$	75	80	22	664
$80 < A \leq 90$	85	90	10	674

Can you complete the last column by using excel and not a calculator?

OGIVE

b. Put the mouse pointer on cell C1, press the left mouse button and drag down to cell C10, release the left mouse button.

c. Then put the mouse pointer on cell E1, hold down the [CTRL] key and again with the left mouse button pressed, drag down to cell E10. Release the [CTRL] key and the mouse.

d. On Excel's Formatting Toolbar, click on the *insert* and then *Chart Wizard button* .

e. Click on *Scatter* icon and choose the *Scatter with straight lines and markers*. You should get the Ogive curve on the screen.

3. Adding chart title, chart axis and data labels

a. Click on the curve

b. On Excel's formatting Toolbar, click on the *layout*, *chart title* and then choose *centered overlay title*.

c. Type the title of the curve.

d. Click on the *axis titles*, *primary horizontal axis title*, then on *title below axis*.

e. Label the horizontal axis.

f. Repeat step d) and e) but this time choose the *primary vertical axis*.

g. Click on *legend* and choose *none* to turn off the legend.

h. To change the scale on your axis, click on *Axis* and change scale.

i. To change the thickness and colour of the line, move the cursor to the line and right click then click on *Format Data Series*.

  1. Click on *line colour* → *solid line* →*colour*

  2. Click on *Line style* → change *width* to your desirable thickness.

HISTOGRAM

Unlike ogives, which have upper limit on the horizontal axis, histograms are made using class interval on the horizontal axis.

a. Put the mouse pointer on cell A1, press the left mouse button and drag down to cell A10, release the left mouse button.

b. Then put the mouse pointer on cell D1, hold down the [CTRL] key and again with the left mouse button pressed, drag down to cell D10. Release the [CTRL] key and the mouse.

c. On Excel's Formatting Toolbar, click on the *insert* and then *Chart Wizard button* .

d. Click on *column* and choose the *clustered column*. You should get the bar graph that on the screen.

e. Follow steps 3(a)-(i) for labelling.

f. To change a Bar graph to Histogram follow the steps below:

  i. Right click on one of the bars, select [Format Data Point],

  ii. Change *series overlap*and *gap width* to 0%.

  iii. On the plot series box click the secondary axis. The bars should be close to each other →Histogram.

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FREQUENCY POLYGON

Frequency polygons are made using midpoints on the horizontal axis.

a. start by selecting the data in B1 through B10 and E1 through E10. Do this in the same way we did for histograms.

b. On Excel's Formatting Toolbar, click on the *insert* and then *Chart Wizard button* .

c. Click on *Scatter* icon and choose the *Scatter with straight lines and markers*. You should get the a frequency polygon curve on the screen.

MATHEMATICAL THINKING WITH FOUNDATION PHASE TEACHERS

Nicky Roberts, Ursula Röntch and Melissa Mentoor

University of Witwatersrand and Kelello Consulting,

The Grove Primary School and Capricorn Primary School

TARGET AUDIENCE:	Foundation Phase teachers
DURATION:	2 hours workshop
MAXIMUM NUMBER OF PARTICIPANTS:	15

ABSTRACT

This workshop will involve Foundation Phase teachers in thinking mathematically by engaging collectively in a few mathematical tasks. By thinking about how we approach and discuss mathematics, we are better able to support our learners. For Foundation Phase teachers this is often difficult: we already know the mathematics which children need to learn; but can't recall how we came to know it. Without this struggle for understanding ourselves, it becomes hard to support learners to come to know the mathematics they require. This workshop will be facilitated to allow space to reflect on what makes our thinking mathematical.

MOTIVATION FOR THE WORKSHOP

Foundation phase teachers have few opportunities to engage in rich mathematical tasks which are accessible at the level of mathematics where they are confident and where they are required to teach. This workshop is intended to provide a space for Foundation Phase teachers to work on appropriate and accessible mathematical tasks which will illicit mathematical discussion with their peers.

DESCRIPTION OF CONTENT OF WORKSHOP

The facilitator will briefly introduce the theoretical rationale for why all mathematics teachers (including those at Foundation Phase) ought to engage themselves in tasks that demand mathematical thinking. This will followed by offering a couple of mathematical tasks which will carefully selected to ensure that they have "high ceilings" but "low thresholds" (acknowledgements to Toni Beardon for this task description phrase). The participants will engage with the tasks, with the facilitator drawing attention and creating time for reflection on what makes their thinking mathematical.

CHILDREN MAKING RESOURCES FOR MEASUREMENT

Ursula Röntch

The Grove Primary School

TARGET AUDIENCE:	Foundation Phase teachers
DURATION:	2-hour workshop slot
MAXIMUM NUMBER OF PARTICIPANTS:	30

ABSTRACT

The measurement content area in the Curriculum and Assessment Policy Statement emphasises informal measurement in Foundation Phase. Measurement includes: time, mass, length, capacity and volume and is revisited each year in each grade. The introduction of formal units is delayed. Why? Many teachers think about measurement as using standard units, but lack the measurement equipment in this school. How can teachers make use of recycled and easily available materials to create measuring instruments which are then used for meaningful mathematics activity and discussion in Foundation Phase classrooms?

MOTIVATION FOR THE WORKSHOP:

Imagining how to engage learners with informal measurement (beyond using their hands for length) can be challenging for teachers. Finding ways to experience measuring and estimating in contexts with few resources can be another difficulty. In this workshop teachers will make some simple but effective instruments for measuring in Foundation Phase classrooms which they will be able to take home with them. These ideas can then be used with the children in their classes.

DESCRIPTION OF CONTENT OF WORKSHOP

The workshop will include:

- Making a meter stick;
- Making a trundle wheel;
- Ideas for overcoming problems with the comparison of mass of objects;
- Informal measures for volume and capacity; as well as area.

MAKING REGRESSION ANALYSIS EASY USING A CASIO SCIENTIFIC CALCULATOR

ASTRID SCHEIBER

CASIO

*Adequate knowledge of calculator skills makes the teaching of Statistics to Grade 12 learners easier and enables the educator to assist their learners more efficiently. This workshop will guide you through Linear Regression Analysis, including finding relationships between variables, the line of best fit and making projections, using the Casio Scientific calculator. Equipment required**: Casio fx-82ZA PLUS** Scientific Calculator.*

TARGET AUDIENCE: Further Education & Training - Mathematics

DURATION: 1 hour

MAXIMUM NUMBER OF PARTICIPANTS: 50

MOTIVATION:

As of 2014, the Grade 12 Statistics syllabus involves the learners making use of available technology to: **[12.10.1 (b)] calculate the linear regression line which best fits a given set of bivariate numerical data** and **[12.10.1 (c)] calculate the correlation co-efficient of a set of bivariate numerical data**. As stated by the current Maths CAPS document. This workshop serves to increase educators understanding of the Casio Scientific calculator. In turn, it will foster self-confidence and a positive attitude towards Statistics, enhancing both the educators and learners understanding of the topic.

CONTENT:

This workshop will cover: Identifying the relationship between bivariate numerical data, inputting bivariate data into the Casio scientific calculator, calculating the correlation co-efficient, finding the equation of the regression line, using TABLE MODE to find the co-ordinates of the line of best fit, calculating projected values - Interpolation & Extrapolation, choosing a random sample of numbers.

Identifying the relationship between bivariate numerical data	5 mins
Inputting bivariate data into the Casio scientific calculator	8 mins
Calculating the correlation co-efficient	8 mins
Finding the equation of the regression line	5 mins
Calculating projected values - Interpolation & Extrapolation	5 mins
Using TABLE MODE to find the co-ordinates of the line of best fit	9 mins
Choosing a random sample of numbers	5 mins
Discussion	15 mins

WORKSHEET:

When we investigate statistical information, we often find there are connections between sets of data. When working with **two variables (x ; y)** this is considered working with bivariate data.

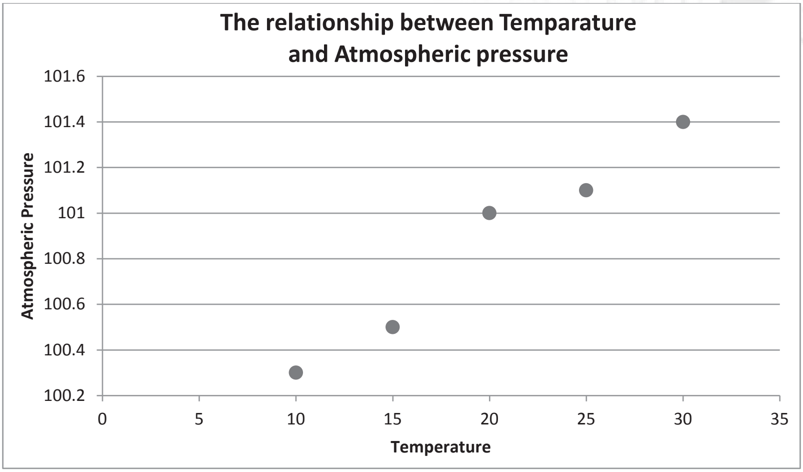
Consider the following table:

Temperature (°C)	Atmospheric pressure (kPa)
10	100,3
15	100,5
20	101,0
25	101,1
30	101,4

Pressure is dependent on Temperature

Hence: temperature is the x variable and pressure the y variable.

We can represent this data by means of a **SCATTER PLOT**. If the data shows a pattern, we can identify whether it forms a linear function or some other type of function such as a Quadratic or Exponential function.



It is clear from the graph that the points tend to form a pattern which resembles a **straight line**

LINEAR REGRESSION

predicts a relationship between a

dependent variable (y) & an independent variable (x),

where the relationship approaches that of a straight line.

$$y = \mathrm{A} + Bx$$

Using the Casio Scientific Calculator:[MODE] [2: STAT]

Key	Menu Item	Explanation
1.	1-VAR	Single variable / Data handling
2.	A + BX	Linear regression
3.	_ + CX^2	Quadratic regression
4.	ln X	Logarithmic regression
5.	e ^ X	Exponential regression
6.	A . B ^ X	AB exponential regression
7.	A . X ^ B	Power regression
8.	1/X	Inverse regression

LINEAR REGRESSION [2: A+BX]

Enter the data into the double variable table
Input the data into the cell where the cursor is located
Use [=] to enter the data items
Input *x*-values first and then *y*-values
Use the [REPLAY] arrows to move the cursor to the *y*-column

	x	*y*
1	10 [=]	100,3 [=]
2	15 [=]	100,5 [=]
3	20 [=]	101 [=]
4	25 [=]	101,1 [=]
5	30 [=]	101,4 [=]

Clear the screen - ready for the STAT sub menu

[AC]

[SHIFT] [1] (STAT)

STAT Linear Regression sub menu

Key	Menu Item	Explanation
5: Reg	1. A 2. B 3. r 4. $\hat{x}$ 5. $\hat{y}$	Regression co-efficient of A Regression co-efficient of B Correlation co-efficient r Estimated value of x Estimated value of y

Correlation is an indication of the relationship between the two variables.

Correlation co-efficient (r) tells us the strength and direction of the correlation.

r lies between -1 and +1 **($-1 \leq r \leq 1$)**

- If r is close to 0, then there is a **weak linear** relationship
- If r is close to -1 or +1, there is a **strong linear** relationship

The sign of r indicates whether the data has a positive or negative correlation (sloping line of best fit)

- Positive correlation

As one quantity increases, the other one increases
As one quantity decreases, the other one decreases

- Negative correlation

As one quantity increases, the other one decreases
As one quantity decreases, the other one increases

CALCULATE THE CORRELATION CO-EFFICIENT

[SHIFT] [1] [5: Reg] [3: r] [=]

r = ...

r is very close to **.......**, hence there is a **...**
correlation between temperature and atmospheric pressure.

Once it is determined that **r > 0,7** (positive or negative) we can calculate the **linear regression line** also called the **line of best fit,** which will help us to predict future values.

$$y = \mathrm{A} + Bx$$

where **A** is the **y-intercept** and **B** is the **gradient/slope**

CALCULATE THE VALUE OF A

[SHIFT] [1] [5: Reg] [1: A] [=]

A = ...

CALCULATE THE VALUE OF B

[SHIFT] [1] [5: Reg] [2: B] [=]

B = ...

So the equation of the line of best fit is

y = ... + ... *x*

PROJECTIONS

<u>CALCULATOR RULE:</u>
Step 1: Input what is given
Step 2: Select which variable is required

Interpolation: the value predicted lies **within** the domain and range of the data set.

Use the line of best fit to estimate the atmospheric pressure when the temperature is 18°C.

18 [SHIFT] [1] [5: Reg] [5: ŷ] [=]

ŷ= ....

The pressure is **............** kPa when the temperature is 18 °C
Extrapolation: the value predicted lies **outside** the domain and range of the data set.
What is the approximate temperature if the atmospheric pressure is 100 kPa?

100 [SHIFT] [1] [5: Reg] [4: $\hat{x}$] [=]

$\hat{x}$= ....

The temperature is **............** °C when the pressure is 100 kPa

TABLE MODE

Using TABLE MODE you can find the **co-ordinates** to plot the **line of best fit.**

[MODE] [3: TABLE]
Enter the equation of the line of best fit
Input a START x-value of 10
Input a END x-value of 30
Input STEPS (INTERVALS) of 5

The co-ordinates to plot are:
(10 ; 100.3) (15 ; 100.58) (20 ; 100.86) (25 ; 101.14) (30 ; 101.42)

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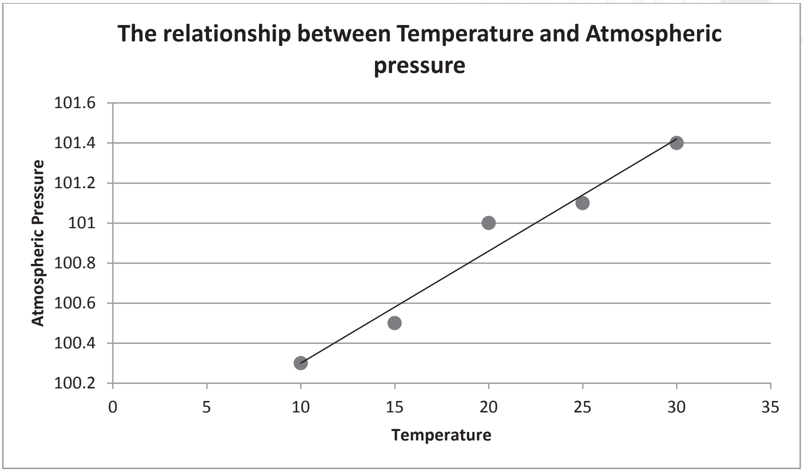

RANDOM INTEGERS

The simplest way to choose a random sample of numbers is to let the calculator do it for you.

Select a random sample of 6 numbers, between 1 and 49, to play the lotto:
[ALPHA] [.] (RanInt) 1 [SHIFT] [)] (,) 49 [)] [=]

NOTE

- every time you use one of these key sequences, you will get a different string of numbers
- Integers are repeated
- This key sequence can be used to flip a coin (1,2)
- This key sequence can be used to throw a die (1,6)

HINTS TO MAKE THE TEACHING OF THE CALCULATOR EASIER

- If you are introducing calculator work to a new class it is easier if all learners have the same calculator.
- Always keep the instruction booklet that you receive with your calculator so that you can refer to it whenever you are not sure how to do a calculation.
- There are 3 modes on the Casio Scientific Calculator fx-82ZA PLUS, always make sure that your calculator is in the right mode before you begin.

[MODE]

1. Computational – normal scientific calculations
2. Statistics – data handling & regression
3. Table – graph work & functions

Calculators play a vital role in the classroom:
not by *substituting* Mathematics, but by *supplementing* our subject.
It's conventional Mathematics by new methods.

REFERENCES:

RADMASTE Centre, ACE – Data Handling and Probability FET (2010) University of the Witwatersrand, SA.

MARC ANCILLOTTI, Data Handling – Scatter plots of bivariate data.

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USING A CALCULATOR TO INVESTIGATE WHETHER A LINEAR, QUADRATIC OR EXPONENTIAL FUNCTION BEST FITS A SET OF BIVARIATE NUMERICAL DATA

Jackie Scheiber

RADMASTE, Wits University

TARGET AUDIENCE: FET Mathematics teachers

DURATION: 2-hours workshop

MAXIMUM NUMBER OF PARTICIPANTS: 30

MOTIVATION:

The Overview of Topics in the GRADE 12 CAPS states:

Represent bivariate numerical data as a scatter plot and suggest intuitively and by simple investigation whether a linear, quadratic or exponential function would best fit the data.

The Casio fx-82ZA PLUS calculator can be used to determine the best function to fit the data.

CONTENT OF THE WORKSHOP:

STEP 1: Revise how to use the Casio fx-82ZA PLUS to determine the linear regression function.

STEP 2: Example 1 – draw a scatter plot and then use the calculator to determine both the linear and exponential regression functions; draw and compare the two regression functions.

STEP 3: Example 2 – draw a scatter plot and then use the calculator to determine both the linear and quadratic regression functions; draw and compare the two regression functions.

STEP 4: Summary of findings and closure.

ABSTRACT:

Grade 12 mathematics learners need to be able to determine whether a linear, quadratic or exponential regression function best fits a set of bivariate data. The learners find this task very easy once they have been shown how to do this on a scientific calculator. In this workshop we will plot two different sets of bivariate data and then use the calculator to determine the regression function that best fits each set of data.

REFERENCE:

Classroom Mathematics Grade 12 (2013)

ANALYSIS OF VAN HIELE'S THEORY IN CIRCLE GEOMETRY: A FOCUS IN FET LEVEL

Sibawu Witness Siyepu
Cape Peninsula University of Technology

TARGET AUDIENCE: FET teachers and subject advisors

DURATION: 2 hours' workshop

MAXIMUM NUMBER OF PARTICIPANTS: 30 participants.

MOTIVATION FOR THE WORKSHOP

Geometry had been known as a problematic section of mathematics especial in South Africa with the teaching approach dominated by teacher tells. This workshop demonstrates the use of van Hiele's theory to enhance teachers' understanding of inductive approach as adopted from Micheal Serra. It is a hands-on enquiry leading participants to develop investigative approach in their teaching of geometry.

DESCRIPTION OF CONTENT OF WORKSHOP

Participants will draw a circle and explore the following concepts he following concepts. 30 minutes

- Radius
- Centre
- Diameter
- Sector
- Annulus
- Secant
- Segment
- Chord
- Tangent
- Arc
- Circumference

Level 2 (Analysis): 30 minutes: Students see figures as collections of properties. They can recognize and name properties of geometric figures, but they do not see relationships between these properties. When describing an object, a student operating at this level might list all the properties the student knows, but not discern which properties are necessary and which are sufficient to describe the object.

Activity 2. Given an economic definition of each concept mentioned in activity 1 above.

The following is a recap of terms that are regularly used when referring to circles.

- Arc: An arc is a part of the circumference of a circle.
- Chord: A chord is a straight line joining the ends of an arc.
- Radius: A radius, r, is any straight line from the centre of the circle to a point on the circumference.
- Diameter: A diameter is a special chord that passes through the centre of the circle. A diameter is the length of a straight line segment from one point on the circumference to another point on the circumference that passes through the centre of the circle.
- Segment: A segment is the part of the circle that is cut off by a chord. A chord divides a circle into two segments.
- Tangent: A tangent is a line that makes contact with a circle at one point on the circumference.

Level 3 (Abstraction): Students perceive relationships between properties and between figures. At this level, students can create meaningful definitions and give informal arguments to justify their reasoning. Logical implications and class inclusions, such as squares being a type of rectangle, are understood. The role and significance of formal deduction, however, is not understood.

Tangent to a Circle: 50 minutes

Draw a tangent to the circle and join the point of tangency with a radius.

Then make a conjecture; what is the relationship between a tangent and a radius of the same circle.

Step 1. Construct a circle with a 4cm radius.

Step 2. Select two points on the circle. Label them A and B.

Step 3. Select a point P on the major arc and construct inscribed $\angle APB$.

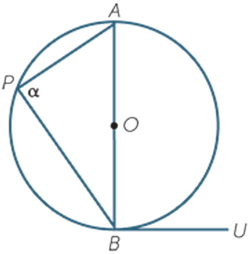
Step 4. With your protractor, measure $\angle APB$.

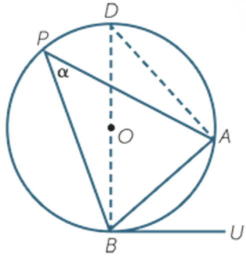
Step 5. Select another point Q on the arc APB and construct inscribed $\angle AQB$.

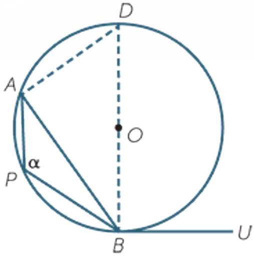
Step 6. Measure $\angle AOB$. How does the measure of $\angle AQB$ compare with the measure of $\angle APB$.

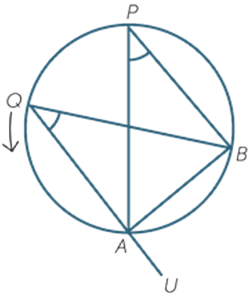
Step 7. Make a conjecture.

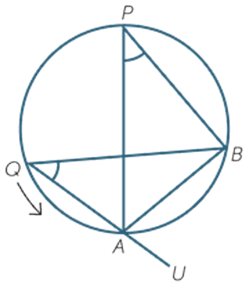
Use the diagrams below to determine the relationship of angles that are equal.

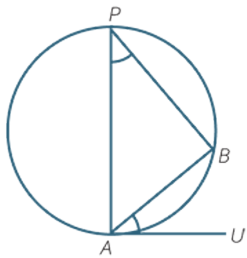












Level 4 (Deduction): Students can construct proofs, understand the role of axioms and definitions, and know the meaning of necessary and sufficient conditions. At this level, students should be able to construct proofs such as those typically found in a high school geometry class.

Wrapping up: 10 minutes

Evaluation of the workshop:

What went well?

What was good?

What can be done differently next time?

Over-view

TEACHING STRATEGIES FOR MENTAL MATHEMATICS (FOUNDATION PHASE)

Connie Skelton
Data Mind

Being able to do calculations in your head is an important life skill and an important part of mathematics. Mental mathematics is also a very important component of the NCS Curriculum and Assessment Policy for Mathematics. The CAPS document lists the number bonds and multiplication table facts that Foundation Phase learners are expected to know and recall for each grade. However, to improve mental calculations, learners need to be taught the most efficient strategies explicitly. This workshop aims to revise the strategies for calculation suggested in the CAPS document and propose some activities that can be used to practise these strategies.

RAPID RECALL CALCULATION FACTS

Tables of the calculation facts for each grade are given below. The list is not exhaustive but includes the key strategies that can be used when developing mental mathematics skills and numeracy. Each year is based on the previous year and teachers should ensure that the knowledge from previous years is revised and carried forward.

This table has been based on the NCS Curriculum and Assessment Policy Statement for Foundation Phase Mathematics.

Grade 1

Grade 1 learners should have	Mental Strategies
Number concept: range 20 • Order a given set of selected numbers. • Compare numbers up to 20 and say which is and more or less	**Number concept: range 20** • Order a given set of selected numbers. • Compare numbers up to 20 and say which is and more or less
Rapid recall:	**Use calculation strategies to add and subtract efficiently:**
Number bonds to 10	Put the larger number first in order to count on or count back
Addition and subtraction facts to 10	Number line
	Doubling and halving
	Building up and breaking down

Table 1: Summary of Grade 1 mental mathematics facts and mental strategies

Grade 2

Grade 2 learners should have	Mental Strategies
Number concept: range 99 • Order a given set of selected numbers. • Compare numbers up to 99 and say which is and more or less	**Number concept: range 99** • Order a given set of selected numbers. • Compare numbers up to 99 and say which is and more or less
Rapid recall:	**Use calculation strategies to add and subtract efficiently:**
Addition and subtraction facts to 20	Put the larger number first in order to count on or count back
Add or subtract multiples of 10 from 0 to 100	Use the relationship between addition and subtraction
	Number line
	Doubling and halving
	Building up and breaking down

Table 2: Summary of Grade 2 mental mathematics facts and mental strategies

Grade 3

Grade 3 learners should have	Mental Strategies
Number concept: range 999 • Order a given set of selected numbers. • Compare numbers up to 1 000 and say which is and more or less	**Number concept: range 1 000** • Order a given set of selected numbers. • Compare numbers up to 1 000 and say which is and more or less
Rapid recall:	**Use calculation strategies to add and subtract efficiently:**
Addition and subtraction facts to 20	Put the larger number first in order to count on or count back
Add or subtract multiples of 10 from 0 to 100	Number line
Multiplication and division facts for the : * two times table up to 2×10 * ten times table up to 10×10	Doubling and halving
	Building up and breaking down
	Use the relationship between addition and subtraction
	Use the relationship between multiplication and division

Table 3: Summary of Grade 3 mental mathematics facts and mental strategies

CHOOSING STRATEGIES

Ten minutes of mental mathematics is recommended every day. This can involve asking learners quick mental starters like: the number before 7 is ...; two more or less than 18 is ....; and 7 + 2; 8 + 2; 9 + 2, etc. These mental mathematics activities can also take the form of printed exercises where learners work independently and write their answers. Peer assessment should be used to mark the answers. Teachers should call the answers out clearly and slowly, write them on the board or display then on a projector.

The teacher should then select two or three learners to explain the strategy that they used to find the answer(s). Discuss the relative merits of different methods with learners. It is important that a safe environment is created in the class so that learners feel confident to discuss their methods. Questions that can be asked include:

- How did you get that answer?
- Is there another way that you could have found the answer?
- Did anyone find the answer in a different way?
- Can you write down the method in a number sentence?

It is important to emphasize that

- learners need efficient and quick methods for mental mathematics
- learners should choose a method that is sensible
- teachers consolidate the main features of the strategies used at the end of the session.

The mental mathematics programme should be developed systematically over the year. As learners cover topics and develop strategies in the main part of the lesson, they can practise them in the mental mathematics programme.

KEY STRATEGIES AND ACTIVITIES

Although a large part of mental mathematics in the Foundation Phase involves the rapid recall of number facts, it is also important to develop these facts to other similar numbers. For example, if a learner knows that 8 + 8 = 16, then the following calculations can be developed from this fact like:

7 + 8; 7 + 18; 8 + 18 and 18 + 18

Different learners will carry out these calculations using different strategies. It is useful that teachers recognise the different strategies that may be forthcoming. This is in turn can be used to build up a range of different strategies and assist learners to choose more efficient strategies where necessary.

Teachers should encourage learners to

- investigate other strategies
- practice other strategies and so build up confidence in using them
- develop their methods that work efficiently.

The NCS CAPS document recommends that it is useful to do mental mathematics with apparatus and to record what is done. The recommended apparatus includes:

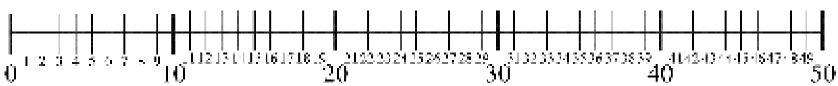
- a number line (structured or empty)
- a number grid
- place value cards (Flard cards)
- counting beads

Paper and pen/pencil can also be used to either write the answers, jot down reminders of patterns or draw images that show how the calculation is being performed.

Once learners have mastered a strategy, they should be encouraged to practise it often enough to build up some speed when answering. Teachers can reinforce a particular strategy and then let learners practise it before the mental mathematics test. They should encourage learners to discuss alternate strategies in these sessions.

Counting on and back

A large number line in class allows learners to appreciate the concept of counting forward and backwards more easily. Remind learners that their rulers are also number lines and can be used for counting.



Counting helps learners develop strategies for calculating. It is also useful for recognising patterns. Learners start counting in ones and then later learn to count in twos, fives, tens and so on. When learners 'skip count', they are actually calculating already.

Learners also need to be able to express numbers verbally, in written number symbols as well as written number words.

Using a number line, learners will also appreciate that when adding two numbers together it is easier to count on from the larger number. This method will eventually be replaced by more efficient methods. When subtracting, learners should count back from the larger number (which will be first).

Example 1: 3 + 6

Think	Do
Start with the largest number	6 + 3
Count on to 9	6 + 1 + 1 + 1 = 9

Example 2: 7 – 2

Think	Do
Start with the largest number	7 – 2
Count back from 7 to 5	7 – 1 – 1 = 5

Developing this strategy further, ask learners to count on or back in twos when the smaller number is an even number, for example 16 – 6 . You can also ask learners to count on in tens when the numbers are multiples of ten, for example for 30 + 60, learners count on in tens from 60 to 90.

Activity 1

Write the biggest number first. Then count on.

1 5 + 17 = □ + Δ = ___

2 1 + 9 = □ + Δ = ___

3 3 + 32 = □ + Δ = ___

4 38 + 6 = □ + Δ = ___

5 18 + 3 = = □ + Δ = ___

6 $28 + 5 = \square + \Delta =$ ___

7 $12 + 17 = \square + \Delta =$ ___

8 $14 + 13 = \square + \Delta =$ ___

9 $38 + 9 = \square + \Delta =$ ___

10 $47 + 19 = \square + \Delta =$ ___

Ordering a given set of numbers

Numbers can be added in any order, so $2 + 5 + 8 = 2 + 8 + 5$. When there are three numbers that need to be added together, two will be added first and the answer then added to the third number. Teach learners to look for pairs of numbers that make 10 and add them first. They should all know the number bonds so they would be looking for $1 + 9$; $2 + 8$; $3 + 7$; $4 + 6$ and $5 + 5$.

For example, $2 + 5 + 8 = (2 + 8) + 5 = 10 + 5 = 15$

It is important that learners realise that they can change the order of the numbers when there are just addition signs, but it gets more complicated when there are minus signs. Order does matter in subtraction!

$9 – 7$ does not equal $7 – 9$.

When there is more than one subtraction, the order can be changed. Teachers should judge the confidence of their group before introducing this technique.

$17 – 5 – 8 = 17 – 8 – 5$

Activity 2

Look for pairs of numbers that make 10. Add these first.

1 $3 + 2 + 8 = (2 + \square) + 3 = 10 + \Delta =$ ___

2 $7 + 4 + 3 = (7 + \square) + 4 = \Delta + 4 =$ ___

3 $9 + 8 + 2 = 9 + (8 + 2) = 9 + \square =$ ___

4 $5 + 5 + 9 = (\square + \Delta) + 9 = 10 + 9 =$ ___

5 $2 + 11 + 8 = 11 + (\square + \Delta) =$ ___

6 $4 + 7 + 3 = \square + (\Delta + 3) =$ ___

7 $2 + 3 + 18 = (\square + \Delta) + 3 =$ ___

8 $19 + 5 + 2 = (\square + \Delta) + \triangle =$ ___

9 $27 + 4 + 3 = (\square + \Delta) + \triangle =$ ___

Note:

- Changing the order of the numbers is a strategy that can only be used if the question is written.
- The examples ask learners to find pairs of numbers that make 10, but other 'easy' numbers like 20 or any other multiple of 10 can be used.

Doubling and halving

Practice doubles up to 20 with learners often so that they attain instant recall of them. Learners often find the doubles the easiest facts to remember. They can be used to:

- simplify calculations
- double one number and halve the other in a product.

Activity 3

Complete.

1 $9 + 9$ is double___

2 $8 + 8 = 8 \times$ ___

3 Half of 14 is ___

4 Half of ___ is 10.

5 $7 +$ ___ is double 7.

6 Half of 18 is ___.

7 10×2 is ______ 10

8 Half of ___ is 50.

Near doubles

Doubles can then be used to add numbers that are close to doubles. For example, learners know that $8 + 8 = 16$, so they can be encouraged to see that $8 + 9$ will be one more than 16.

As learners become more confident, give them numbers that are two and then three apart to use to practice this strategy.

Activity 4

1 $3 + 4 = (3 + 3) + 1 =$ ___

2 $6 + 5 = (6 + 6) – 1$ ___ = ___

3 $9 + 10 = (9 + 9) + 1 =$ ___

4 $9 + 8 = (9 + 9) –$ ___ = ___

5 $10 + 11 = (10 +$ ___ $) + 1 =$ ___

Activity 5

	Number	Double	Double + 1	Double + 2
1	4	4 + 4 = 8	4 + 5 = 9	4 + 6 = 10
2	2	2 + 2 = ___	2 + __ = ___	2 + 4 = ___
3	5			
4	15			
5	11			

Change a number to 10 and then subtract or add 1

This strategy is useful for adding numbers that are close to a multiple of 10. When numbers are close to 10 or a multiple of 10, the number to be added can be broken into a multiple of 10 plus a small number or a multiple of 10 minus a small number.

Example: 8 + 9

Think	Do
9 can be written as 10 – 1	8 + 10 – 1
Add 10 to 8 and then subtract 1	18 – 1 = 17

Example: 6 + 11

Think	Do
11 can be written as 10 + 1	6 + 10 + 1
Add 10 to 6 and then add 1	16 + 1 = 17

Activity 6

Complete.

1 $6 + 9 = 6 + 10 - 1 =$ ___

2 $8 + 9 = 8 + 10 +$ ___ = ___

3 $11 + 7 = 10 + 7 - 1 =$ ___

4 $14 + 9 = 14 + 10 -$ ___ = ___

5 $21 + 7 = 20 + 1 + 7 =$ ___

Building up and breaking down numbers

Revise place value using Flard cards or calculators. Learners should be able to break down numbers like 247 = 200 + 40 + 7. This is a useful strategy for adding and subtracting. Both numbers can be broken down like this, but it may be quicker to just break one of the numbers.

Example: 8 + 12

Think	Do
Leave the first number and break down the second number.	8 + (10 + 2)
Add 8 + 10 and count on by 2	18 + 2 = 20

Example: 48 – 30

Think	Do
Break down the number that has tens and units	40 + 8 – 30
Rearrange.	40 – 30 + 8
Subtract 30 and count on 8	10 + 8 = 18

Use the relationship between addition and subtraction

Every addition calculation can be replaced by an equivalent subtraction calculation and similarly every subtraction can be replaced by an addition. For example with addition,

$$7 + 11 = 18$$

$$7 = 18 - 11$$

$$11 = 18 - 7$$

For subtraction,

$$20 - 6 = 14$$

$$20 = 14 + 6$$

$$6 = 20 - 14$$

Activity 7

Complete.

1 $12 + \square = 18$ and $18 - \square = 12$

2 $10 - 0 = \square$ and $\square + 0 = 10$

3 $19 - \square = 10$ and $19 - 10 = \square$

4 $\square + 12 = 15$ and $15 - \square = 12$

5 $14 - 9 = \square$ and $14 - \square = 9$

6 $6 + \square = 20$ and $20 - 6 = \square$

7 $15 + \square = 19$ and $19 - 15 = \square$

8 $9 - 4 = \square$ and $\square - 5 = 4$

9 $9 + \square = 16$ and $16 - 7 = \square$

10 $19 + 17 = \square$ and $\square - 19 = 16$

Use relationship between multiplication and division

Grouping actual physical objects and numbers is essential for multiplication and division. Multiplication is usually where objects are grouped together and is associated with repeated addition and doubling. Division is associated with breaking up a number of objects into equal groupings, repeated subtraction and halving. Teachers should give learners a lot of activities that involve forward and backward skip counting.

Division and multiplication are inverse operations. Any two whole numbers can be multiplied to get another whole number, but this is not the same for division. It is a very useful strategy for learners to know which of the numbers will result in a whole number when dividing.

Grade 3 learners must be able to rapidly recall multiplication and division facts for the two and ten times tables.

This is a gradual process and learners in Grade 1 and 2 can be encouraged to start by doing the following exercises:

1 Skip count in twos

2 Skip count in tens

Grade 3 learners should:

1 recall the two times table up to 2×10

2 recall the ten times table up to 10×10

3 recall division facts for the two and 10 times table.

For every multiplication there is a division sum and vice versa.

$$7 \times 10 = 70$$

$$7 = 70 \div 10$$

$$10 = 70 \div 7$$

For division,

$$20 \div 2 = 10$$

$$20 = 10 \times 2$$

Activity 8

Complete the empty blocks.

		× 2	×10	÷2	double	half	÷10
1	4						
2	10						
3	20						
4	14						
5	7						
6	17						
7	19						

ESTIMATION

Encourage estimations and checking answers throughout the year.

Estimating is the ability to make reasonable guesses about a quantity. In the lower grades, learners deal with estimations informally and do not learn to round off. It is however important that these younger learners gain experience with estimation and comparing whether their estimate is larger or smaller than the actual count. They need to be able to look at a group of up to 20 objects and have a good sense of whether there are about 5, 10, 15 or 20 objects.

CONCLUSION

The strategies listed above should be practised throughout the year in a structured mental mathematics programme. Learners can also play mathematical games to practise and memorise number facts. Calculators can be used to enhance the understanding of the strategies, but they should not be used during mental mathematics tests.

Mental mathematics is one of the most important tools for learning mathematics. It not only means to calculate quickly, but involves conceptual understanding and problem solving.

TEACHING STRATEGIES FOR MENTAL MATHEMATICS (INTERMEDIATE PHASE)

Connie Skelton

Data Mind

Mental mathematics is one of the most important tools for learning mathematics. It not only means to calculate quickly, but involves conceptual understanding and problem solving. Mental mathematics is also a very useful life skill. It is also a very important component of the NCS Curriculum and Assessment Policy for Mathematics. The CAPS document lists the bonds, number facts and tables that Intermediate Phase learners are expected to know and recall for each grade. However, to improve mental calculations, learners need to be taught the most efficient strategies explicitly. This workshop aims to revise the strategies for calculation suggested in the CAPS document and propose some activities that can be used to practise these strategies.

RAPID RECALL CALCULATION FACTS

Tables of the calculation facts for each grade are given below. The list is not exhaustive but includes the key strategies that can be used when developing mental mathematics skills. Each year is based on the previous year and teachers should ensure that the knowledge from previous years is revised and carried forward.

This table has been based on the NCS Curriculum and Assessment Policy Statement for Intermediate Phase Mathematics.

Grade 4

Grade 4 learners should have	Mental Strategies
Rapid recall of number facts for • number bonds: addition and subtraction facts for: ◊ units ◊ multiples of 10, 100 and 1 000 • times tables involving multiplication of whole numbers to at least 10 x 10 **Rapid recall of multiplication facts for** • units by multiples of 10 • units by multiples of 100 **Number range for multiples and factors** Multiples of 1-digit numbers to at least 100	**Calculation techniques** • count forwards and backwards in 2s, 3s, 5s, 10s, 25s, 50s, between 0 and at least 10 000 • count forwards and backwards in 100s between 0 and at least 1 000 • Ordering and comparing up to 4-digit numbers • Represent odd and even numbers to at least 1 000 • doubling and halving, • using a number line, • using multiplication to do division, • multiplying by 10 and 100 • multiplying by multiples of 10 and 100 • dividing by 10, 100 and 1 000 • recognize the place value of digits in whole numbers to at least 4-digit numbers • Round off to the nearest 10, 100, 1 000 • estimation • rounding off to the nearest 10 and compensating • building up and breaking down numbers • adding and subtracting units, multiples of 10 and multiples of 100 to/from any 3-digit number • using the inverse relationship between addition and subtraction • using multiplication and division as inverse operations. • Recognize and use the commutative; associative; and distributive properties of whole numbers.

Table 1: Summary of Grade 4 mental mathematics facts and mental strategies

Grade 5

Grade 5 learners should have	Mental Strategies
Rapid recall of number facts for • number bonds: addition and subtraction facts for: ◊ units ◊ multiples of 10, 100, 1 000 and 10 000 • times tables involving multiplication of whole numbers to at least 10 x 10 **Rapid recall of multiplication facts for** • units by multiples of 10, 100, 1 000, 10 000 **Number range for multiples and factors** Multiples of 2-digit whole numbers to at least 100 Factors of 2-digit whole numbers to at least 100	**Calculation techniques** • count forwards and backwards in whole number intervals up to at least 10 000 • Ordering and comparing up to 6-digit numbers • Represent odd and even numbers to at least 1 000 • multiples • factors • doubling and halving, • using a number line, • using multiplication to do division, • multiplying by 10, 100 and 1 000 • multiplying by multiples of 10 and 100 • dividing by 10, 100 and 1 000 • recognize the place value of digits in whole numbers to at least 6-digit numbers • Round off to the nearest 5, 10, 100, 1 000 • estimation • rounding off to the nearest 10, 100 and 1 000 and compensating • building up and breaking down numbers • adding and subtracting units, multiples of 10, 100 and 1 000 to/from any 5-digit number • using the inverse relationship between addition and subtraction • using multiplication and division as inverse operations. • Recognize and use the commutative; associative; and distributive properties of whole numbers. • adding and subtracting in columns • 0 in terms of its additive property • 1 in terms of its multiplicative property

Table 2: Summary of Grade 5 mental mathematics facts and mental strategies

Grade 6

Grade 6 learners should have	Mental Strategies
Rapid recall of number facts for • number bonds: addition and subtraction facts for: ◊ units ◊ multiples of 10, 100, 1 000 and 10 000 • times tables involving multiplication of whole numbers to at least 12 x 12 **Rapid recall of multiplication facts for** • units by multiples of 10, 100, 1 000, 10 000 **Number range for multiples and factors** Multiples of 2- and 3-digit whole numbers to at least 100 Factors of 2- and 3-digit whole numbers to at least 100 Prime factors of numbers to at least 100	**Calculation techniques** • count forwards and backwards in whole number intervals up to at least 10 000 • Ordering and comparing up to 9-digit numbers • Represent prime numbers to at least 100 • Represent odd and even numbers to at least 1 000 • multiples • factors • doubling and halving, • using a number line, • using multiplication to do division, • multiplying by 10, 100 and 1 000 • multiplying by multiples of 10 and 100 • dividing by 10, 100 and 1 000 • recognize the place value of digits in whole numbers to at least 9-digit numbers • Round off to the nearest 5, 10, 100, 1 000, 10 000, 100 000 and 1 000 000 • estimation • rounding off to the nearest 10, 100 and 1 000 and compensating • building up and breaking down numbers • adding and subtracting units, multiples of 10, 100 and 1 000 to/from any 5-digit number • using the inverse relationship between addition and subtraction • using multiplication and division as inverse operations. • Recognize and use the commutative; associative; and distributive properties of whole numbers. • adding, subtracting and multiplying in columns • long division • 0 in terms of its additive property • 1 in terms of its multiplicative property

Table 3: Summary of Grade 6 mental mathematics facts and mental strategies

CHOOSING STRATEGIES

It is important to emphasize that

- learners need efficient and quick methods for mental mathematics
- learners should choose a method that is sensible
- teachers consolidate the main features of the strategies used at the end of the session.

KEY STRATEGIES AND ACTIVITIES

- Counting
- Ordering a given set of numbers
- Adding three or four small numbers by finding pairs that make 10
- Doubling and halving
- Near doubles
- Place value (partitioning using multiples of 10, 100 and 1 000)
- Change a number to a multiple of 10 and then subtract or add 1, 2 or 3 (compensating)
- Building up and breaking down numbers
- Use the relationship between addition and subtraction
- Use the relationship between multiplication and division

Counting

Learners learn to count by beginning at one and then counting on in ones. Although it is important for learners to do this counting as a number-based exercise, it is important that they also be reminded that counting can be object-based and context-based (candles on a cake). Give learners opportunities to count real objects like

- all the chairs in the classroom
- the number of pencils in a jar
- the number of boys/girls in a group
- the number of shoes in a class and ask them why it may be more useful to count the shoes in twos
- Ask learners to hold one hand in the air with five fingers spread out. What would they count in if they counted all the fingers?
- Ask learners when counting in tens, 50s or 100s would be useful.

Number lines can be useful to help learners count forwards and backwards. Empty number lines can be used for single digit numbers, e.g. 4 + 9.

Encourage learners to see that the second number line may be more efficient than the first. They should start from the bigger number and count on.

Another way of using the empty number line could be to count on one to 10 and then count on another 3.

We can also use counting to find the multiples of a number, for example.

1 2; 4; 6; 8; ___; ___; ___

2 3; 6; 9; 12; ___; ___; ___

3 10; 20; 30; 40; ___; ___; ___

4 100; 200; 300; ___; ___; ___; 700

In general, counting helps learners develop strategies for calculating. Learners are calculating when they 'skip count'.

Ordering and reordering (commutative property)

When learners order numbers they use an important aspect of place value. Numbers with more digits will be larger than those with fewer digits. The smallest 2-digit number (10) is larger than the biggest 1-digit number (9) and so on. If the numbers that need to be ordered have the same number of digits, learners start by comparing the digits in each place value starting from the left.

Some concrete practice can be to take 10 to 15 cards with numbers on them. Ask learners to shuffle them and to arrange the cards from smallest to biggest. You can also remove some cards and then ask learners to replace them in order; work out how many cards there are between 63 and 75 and then count the cards between 63 and 75. Are the two numbers the same? Why or why not?

Number lines can also be used to compare numbers. Remember to revise words like ‘before’, ‘after’ and ‘between’.

	Number	comes before	comes after	+1	–1
1	11				
2	199				
3	935				
4	60				
5	174				

Sometimes it is easier to do a calculation by changing the order of the numbers (Commutative Law). For example

Look for pairs of numbers that make 10 or 20 and use these first.

1 $5 + 7 + 3 = 5 + (\_\_\_ + 3) = \_\_\_$

2 $4 + 7 + 6 = (4 + 6) + 7 = \_\_\_$

3 $3 + 18 + 2 = 3 + (18 + \_\_\_) = \_\_\_$

It is important that learners realise that they can change the order of the numbers when there are just addition signs, but it gets more complicated when there are minus signs. Order does matter in subtraction!

$9 – 7$ does not equal $7 – 9$.

When there is more than one subtraction, the order can be changed. Teachers should judge the confidence of their group before introducing this technique.

$17 – 5 – 8 = 17 – 8 – 5$

Doubling and halving

Practice doubles with learners often so that they attain instant recall of them. Learners often find the doubles the easiest facts to remember. They can be used to:

- simplify calculations
- double one number and halve the other in a product.

Double	8	12	15	16	22	50
Halve	20	24	18	16	15	90

Near doubles

Doubles can then be used to add numbers that are close to doubles. For example, learners know that 18 + 18 = 36, so they can be encouraged to see that 18 + 19 will be one more than 36.

As learners become more confident, give them numbers that are two and then three apart to use to practice this strategy.

1 3 + 4 = (3 + 3) + 1 = ___

2 16 + 15 = (15 + 15) + 1 = ___

3 19 + 20 = (19 + ___) + 1 = ___

4 30 + 29 = (30 + ___) – ___ = ___

5 100 + 102 = (10 0+ ___) + ___ = ___

	Number	Double	Double + 1	Double + 2
1	4	4 + 4 = 8	4 + 5 = 9	4 + 6 = 10
2	12	12 + 12 = ___	12 + __ = ___	12 + 14 = ___
3	25			

Place value

Learners should be able to break down numbers like 957 = 900 + 50 + 7.

		1 000	100	10	1
1	542		5	4	2
2	2 118				
3	6 261				

This is a useful strategy for adding and subtracting. Both numbers can be broken down like this, but it may be quicker to just break one of the numbers.

Example: 7 + 32

Think	**Do**
Leave the first number and break down the second number.	7 + (30 + 2)
Add 7 + 30 and count on by 6	37 + 2 = 39

Example: 48 – 30

Think	Do
Break down the number that has tens and units	40 + 8 – 30
Rearrange.	40 – 30 + 8
Subtract 30 and count on 8	10 + 8 = 18

Break down the numbers and add or subtract.

1 $32 + 47 = (30 + 2) + (40 + 7) =$ ___

2 $19 + 15 =$ (___ + ___) $+ (10 + 5) =$ ___

3 $99 - 84 = (90 +$ ___$) - (80 +$ ___$) =$ ___

Change a number to a multiple of 10 and then subtract or add 1, 2 or 3 (compensating)

It is quite useful when doing mental mathematics to be able to recognise that one of the numbers is close to 10 or a multiple of 10. The other number is then used to make up the 10 by breaking it down.

$17 + 8 = 17 + 3 + 5 = 20 + 5 = 25$

$57 + 15 = 57 + 3 + 12 =$ ___ $+ 12 =$ ___

$48 - 24 =$ ___ $- 8 - 16 =$ ___

$5{,}9 + 2{,}4 =$ ___ + ___ + ___ = ___

This is similar to the way change used to be counted back. If the item cost R37 and you paid with a R50 note, the change would start at R37 and then R3 + R10 = R13 would be added to get R50.

Use the relationship between addition and subtraction

Every addition calculation can be replaced by an equivalent subtraction calculation and similarly every subtraction can be replaced by an addition. For example with addition,

$$17 + 12 = 29$$

$$17 = 29 - 12$$

$$12 = 29 - 17$$

For subtraction,

$$38 - 16 = 22$$
$$38 = 16 + 22$$
$$16 = 38 - 22$$

1 $7 + 9 = \_\_\_$ and $16 - 9 = \_\_\_$ and $\_\_\_ - 7 = 9$

2 $19 + 3 = \_\_\_$ and $\_\_\_ - 19 = \_\_\_$ and $\_\_\_ - 3 = 19$

3 $\_\_\_ + 4 = 67$ and $67 - 4 = \_\_\_$ and $67 - \_\_\_ = 4$

4 $27 - \_\_\_ = 13$ and $13 + 14 = \_\_\_$ and $\_\_\_ - 13 = 14$

Multiplication and division

One of the key factors in learners developing mental mathematics skills is the instant recall of multiplication and division facts. In the Intermediate Phase, the Grade 4 and 5 learners should know their multiplication and division tables up to 10 x 10 and in Grade 6 to 12 x 12. This takes time and practice.

Skip counting can be used to work out that 5 x 3 → 3; 6; 9; 12; **15**

The commutative property makes multiplying 25 x 4 easier by saying 4 x 25 → 25; 50; 75; 100.

Learners can also reorder numbers to calculate ‘easier’ facts first, for example:

$4 \times 6 \times 5 = (4 \times 5) \times 6 = 20 \times 6 = 120$

$12 \times 9 = 12 \times 10 - 12 = 120 - 12 = 108$

Doubling and halving can be used to find easier numbers to work with, for example 16 x 5 = 8 x 10 = 80 (halve 16 and double 5)

Every multiplication calculation can be replaced by an equivalent division calculation and similarly every division can be replaced by a multiplication.

$30 \times 7 = \_\_\_$

$\_\_\_ \div 7 = 30$

$\_\_\_ \div 30 = 7$

$7 \times 30 = \_\_\_$

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ESTIMATION

Encourage estimations and checking answers throughout the year. Estimating is the ability to make reasonable guesses about a quantity. In the lower grades, learners deal with estimations informally and do not learn to round off.

Estimating is also very useful before starting a calculation. If the calculation is 28×42, learners should be taught to round the numbers to the nearest multiple of ten and get an estimate of the answer. For example, $28 \times 42 \approx 30 \times 40 \approx 1\,200$ (actual answer 1 176).

By the end of Grade 6 learners should be able to round off to 5, 10, 100, 1 000, 10 000, 100 000 and 1 000 000.

CONCLUSION

The strategies listed above should be practised throughout the year in a structured mental mathematics programme. Learners can also play mathematical games to practise and memorise number facts. Calculators can be used to enhance the understanding of the strategies, but they should not be used during mental mathematics tests.

Mental mathematics is one of the most important tools for learning mathematics. It not only means to calculate quickly, but involves conceptual understanding and problem solving.

USEFUL WEBPAGES

http://www.compare4kids.co.uk/mental-maths.php

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PROBABILITY

Thabisa Tikolo
STATISTICS SOUTH AFRICA

Probability is a topic that some educators tend to struggle with and thus avoid teaching it to learners. This is an indication that teachers are not yet completely comfortable with the topic. This workshop is an attempt to develop teachers' content knowledge and understanding of Probability in grades 7 -9 as it forms the basis for all the Probability concepts that they will be dealing with in the FET phase- both in Mathematics and Maths Lit.
In this workshop relevant Data from both Census@Schools 2009 and Census 2011 will be used

PROBABILITY IN THE GET PHASE

Relative frequency and probability
During this workshop teachers will: List all the possible outcomes of simple experiments, determine the relative frequency of actual outcomes for a series of trials, Determine the probability of outcomes of simple situations using the definition for probability, predict the frequency of possible outcomes based on their probability, Compare relative frequency and probability and explain possible differences, determine probabilities for compound events using two-way tables and tree diagrams
A complete memo with all the answers to the examples from the hand out will be provided to every participant at the end of the workshop.

Proposed time allocation for workshop activities:

1. List all the possible outcomes of simple experiments	20 minutes
2. Determine the relative frequency of actual outcomes for a series of trials	20 minutes
3.Determine the probability of outcomes of simple situations using the definition for probability	20 minutes
4. Predict the frequency of possible outcomes based on their probability.	20 minutes
5. Compare relative frequency and probability and explain possible differences.	20 minutes
6. Determine probabilities for compound events using two-way tables and tree diagrams	20 minutes

ACTIVITY 1 – RECAP

Probability– is the part of mathematics that studies chance or likelihood. For example, when you see on the weather report that there is a 60% chance of rain, they are talking about a probability.

1.1 Explanation of terms:

Experiment – one or more activities to see what happens e.g. tossing a coin, throwing a dice is an activity or experiment.

Sample Space – made up of all possible outcomes of an experiment. E.g. Sample space of tossing a coin = S = {H; T}

Event - An outcome or combination of outcomes that we are interested in e.g. getting tails when tossing a coin is an event.

Favourable outcomes – outcome or combination of outcomes of an event that we are interested in

1.2 Probability Scales

Rather than using words to describe the chance of an event happening, you can give probability as a number **between 0 and 1**.

- This number can be written as a fraction, percentage or decimal
- If it is **impossible** for an event to happen, the probability is **0.**
- If an event is **certain** to happen, the probability is **1**.
- All other probabilities are **greater than 0, but less than 1**.

Examples:

Some events **always happen**. We say that they are certain to happen and give them a probability of **1**.

- It is **certain** that the day after Monday is Tuesday
- The probability that the day after Monday is Tuesday is **1**.
- Some events **never happen**. We say that they are **impossible** and give them a probability of **0**.
- If you throw an ordinary dice, it is impossible to get a 7?
- The probability of getting a 7 when you throw an ordinary dice is 0.
- Some events are **not certain**, but are **not impossible either**. They may or may not happen. These probabilities lie **between 0 and 1**.
- If you toss a fair coin it may land on heads or it may not. The chances are **equally likely**.

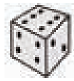
1.3 ACTIVITY: LISTING OUTCOMES OF AN EXPERIMENT:

- If we do a probability experiment, for example throw a die, toss a coin, spin a spinner, we can list all the possible outcomes.
- An event is a particular outcome or group of outcomes.
- Favorable outcomes are the outcomes which give the event you are interested in.

- If the die is fair, then we can say that the outcomes are equally likely. In other words all numbers have the same chance of being thrown. One number doesn't have a greater chance than the others of being thrown.
- If a die is NOT fair, we say that it is **biased**.

Exercise 1 A

Suppose you throw a fair die**.**



1) List all the possible outcomes
2) List a favorable outcome for the event:
a) Getting a 4
b) Getting an even number

Exercise 1.B

An eight-sided die (like the one alongside) is thrown

a) List all the possible outcomes.
b) List all the favourable outcomes for the following events:
Event A: Getting a 2
Event B: Getting an odd number
Event C: Getting a number bigger than 4.

Exercise 1.C

A coin is randomly taken from this purse.
Inside the purse are a R5 coin, a R2 coin, two R1 coins, and a 20c coin.

a) List all the possible outcomes.
b) List all the favourable outcomes for the following events:
Event D: Getting a R1 coin
Event E: Getting a 'bronze' coin
Event F: Getting a coin worth more than R4.

ACTIVITY2. FINDING THE RELATIVE FREQUENCY:

We find the relative frequency of an experiment by performing the experiment or by collecting information from past records

Relative frequency can be calculated using the following formulae:

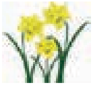
$$\textit{Relative frequency} = \frac{\text{no.of times an outcome occurs in an experiment}}{\text{total number of trials in the experiment}}$$

OR

$$\textit{Relative Frequency} = \frac{\text{no.of times an outcome occurs in a survey}}{\text{total number of observations in a survey}}$$

Example:
In an experiment a drawing pin is dropped 100 times.
It lands with its **point up** 37 times.
What is the relative frequency of the drawing pin landing point up?
Solution:
Relative frequency $== \frac{\textit{no.of times the drawing pin lends point up}}{\textit{total number of times the drawing pin is dropped}} = \frac{37}{100}$
= 0, 37 or 37%

Exercise 2.1
1) In an experiment a gardener planted 40 daffodil bulbs.



36 of these daffodil bulbs produced flowers.
Use these results to find the relative frequency that a daffodil bulb will produce a flower. Use the formula:
Relative frequency $= \frac{\textit{no.of bulbs producing flowers}}{\textit{total number of bulbs planted}}$

Exercise 2.2

Thandi keeps a record of her chess games with Helen. Out of the first 30 games, Helen wins 21 games.

a) Use the results to work out
   (i) The relative frequency that Helen wins
   (ii) Use the two relative frequencies to predict whether Thandi will win her next game of chess with Helen.

ACTIVITY3. CALCULATING PROBABILITY:
In the previous section we looked at how you could estimate probabilities by doing experiments or making observations.
When we have a situations where each outcome is **equally likely** to occur, we can calculate the **probability** of a particular outcome occurring.
Probability can be calculated using the following formula:
Probability $= \frac{\textit{no.of favourable outcomes in an event}}{\textit{total number of possible outcomes}}$

Example:
An ordinary die is rolled. What is the probability of getting?
a) A 6?
b) An odd number?
c) A 2 or 3?

Solution:
The possible outcomes on an ordinary die are: 1; 2; 3; 4; 5; and 6.
The total number of possible outcomes is 6.
a) The favourable outcome is 6.
    The number of favourable outcomes is 1
P (6) = $\frac{1}{6}$ $\approx 0, 17 \approx 16, 7\%$
b) The favourable outcomes are 1; 3 and 5.
    The number of favourable outcomes is 3
P (odd) = $\frac{3}{6} = \frac{1}{2} \approx 0, 5 \approx 50\%$
c) The favourable outcomes are 2 and 3.
    The number of favourable outcomes is 2
P (2or 3) = $\frac{2}{6} = \frac{1}{3} \approx 0, 33 \approx 3, 33\%$
Exercise: 3.1
1) A bag contains a red counter, a blue counter and a green counter.
A counter is taken from the bag at random.
a) What are the possible outcomes?
b) What is the probability of taking?
(i) A red counter?
(ii) A red or green counter?
(iii) A counter that is not blue?
(iv) A yellow counter?
2) A card is randomly taken from a full pack of 52 playing cards with no jokers

- Thirteen of the cards are marked with a black spade (♠)
- Thirteen of the cards are marked with a red heart (♥)
- Thirteen of the cards are marked with a red diamond (♦)
- Thirteen of the cards are marked with a black club (♣)
- Each set of thirteen cards is called a **suit**
- The 1 is generally marked A and is called the Ace
- The picture cards are **Jack, Queen** and **King**.

What is the probability that the card that is taken is:

a) A red card?
b) A heart?
c) The Ace of hearts?

NOTE:

When a question asks you to estimate the probability, it is actually asking you to calculate the relative frequency

The larger the number of trials or observations, the closer the relative frequency is to the probability.

ACTIVITY4. PREDICTING FREQUENCIES BASED ON PROBABILITY

We can use the probability of an outcome occurring to predict or estimate the frequency of that outcome occurring in an experiment.

We can use the formula to estimate the frequency of an outcome occurring:

Predicted frequency of an outcome = *Probability of that outcome occurring* × *Number of repeats of the experiment*

Example

A fair coin is tossed 10 times. How many heads could you expect to get?

Solution:

The probability of getting heads = $\frac{1}{2}$

Predicted frequency of heads = probability of getting a head × number of times the coin is tossed

$\frac{1}{2}$ X 10 = 5

Exercise 4.1

1) There are 50 cars in a school car park. Five of the cars are black.
a) What is the probability that the first car to leave the car park will be black?
b) If the probability that a red car is the first to leave is 0, 2, how many red cars are there in the car park?

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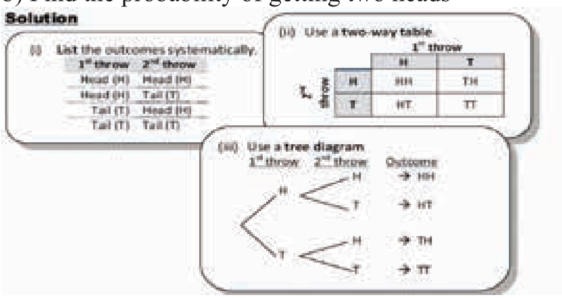
Exercise 4.2
500 tickets are sold for a prize draw.
a) Sam buys one ticket. What is the probability that he wins the first prize?
b) The probability of Cynthia winning first prize is $\frac{1}{20}$. How many tickets did she buy?

ACTIVITY5. PROBABILITIES OF COMPOUND EVENTS OCCURRING:
So far, we have been looking at probabilities of single events occurring.
In some problems, we will have to find the probability of compound events occurring. Events are called compound events when two (or more) activities take place.
Examples of compound events:

- We spin a spinner and then select a card.
- We throw two dice together
- We toss a coin twice.

In situations like these, we can find all the possible outcomes using:
(i) A list
(ii) A two-way table
(iii) A tree diagram.

Example
A fair coin is tossed twice.
a) Identify all the possible outcomes
b) Find the probability of getting two heads



a) All three of these methods show that there are four possible outcomes, namely: HH, HT, TH and TT.
b) There is only one favourable outcome in the event of getting two heads, so:
P (H; H) $= \frac{1}{4}$

Exercise 5.1

1) A fair coin is tossed and an ordinary die is rolled

   a) Copy and complete the two-way table to list all the possible outcomes

		Die					
		1	2	3	4	5	6
Coin	H						
	T						

b) Use the table to calculate the probability of getting:
(i) A head and 5?
(ii) A tail and 6?
(iii) A tail and an even number?
(iv) A tail and an odd number?
(v) A head and a number greater than 3?
(vi) An odd number

Exercise .5.2

Mbali has 3 pairs of jeans: one blue, one black and one red. She also has 4 shirts: one blue, one white, one black and one striped.

a) Draw a tree diagram to show how many outfits she can 'mix and match' with these clothes

b) If she chose a shirt and a pair of jeans with her eyes shut what is the probability that she would choose the black jeans and the black shirt?

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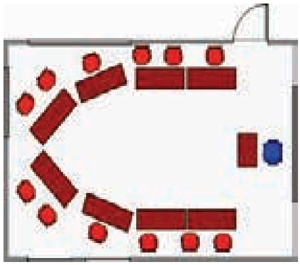
Speed B. (1997). Higher Mathematics for GCSE. Collins Educational

DEVELOPMENT OF A SECURE NUMBER SENSE

Mosadi Kruger

Department of Education - Frances Baard

Desk setting (Different development)



Mat setting

(Same development)

TARGET AUDIENCE: Foundation Phase teachers

DURATION: 2 hours

MAXIMUM NO. OF PARTICIPANTS: 30

GROUP TEACHING AND GROUP WORK

- Teaching is more effective when learners are taught in small groups
- Small group teaching must be according to learners' development stages
- **Numbers, Operations** and **Relationships** and **patterns** should be the focus with activities such as **counting**, **manipulating** of numbers and **problem solving**
- These two content areas cannot be separated as number patterns helps develop learner's strong number sense.
- 2 groups can be seen in a day, thus suggest group teaching can be done Monday to Thursday
- Give learners opportunity to **do**, **talk** and **record** their mathematical thinking during small group teaching
- Do not **tell** them what to do but **allow** them to **think** and **give their own** solutions to given word problem

- Friday can be set aside for practical activities such as **space and shape**, **measurement** and **data handling** which will be 1 hour and these activities need to be thoroughly planned for
- Always consider the weighting of the content areas when teaching and setting formal tasks
- Make sure to manage time accordingly

CLASSROOM ORGANISATION

- Set rules and routines so that learners learn self-discipline and take responsibility for their own learning.
- Work with **2** groups on the mat per day.
- Have independent activities for seat work to reinforce what was taught on the mat.
- Give learners activities they can handle to avoid interruptions during group teaching.
- Plan and know exactly what outcome you are expecting for each group daily/weekly.
- ONLY **one** lesson plan planned for the whole class but **differentiated** activities**.**
- Sit learners with their mixed groups at their desks to minimize noise.
- Introduces same concept **orally** and **practically**, consider the number ranges each groups is at.
- Deal with a group of about +/-10 learners on the mat.
- Teach learners of same **developmental** stages so that they are taught at a pace that is comfortable for them and their learning is scaffolded.
- The weaker learners can benefit from more time, support and attention in a small group situation.
- No group of developmental stage will be neglected.
- Quicker learners can help weaker learners during independent activities.

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ROTATION OF GROUPS

Teachers can work with 2 small groups a day on the mat

MONDAY	*TUESDAY*	*WEDNESDAY*	*THURSDAY*	*FRIDAY*
GROUPS **Red** **&** **Blue**	**GROUPS** **Yellow** **&** **green**	**GROUPS** **Red** **&** **Blue**	**GROUPS** **Yellow** **&** **green**	**WHOLE CLASS** **Practical Activities**

THREE KEY MAT ACTIVITIES (Practical activities)

MAT ACTIVITIES (These activity must happen always on the mat)

(1) **Counting Activity**: **(see 1.1 & 1.2: CAPS)**

Counting **all** objects

Counting **out a given number** of objects

Estimation, count on and group counting **5**s, **2**s, **10**s, etc.

Estimation and group counting of **20**s, **25**s, **50**s and **100**s.

(2) **Manipulating numbers**:

Mental Maths activities: developing a mental number line (**Oral**).

Children visualize numbers in a sequence, moving forward and backward the number ladder with small steps **(see 1.16: CAPS).**

Recognise, identify, read and write number symbols and number names **(see 1.3: CAPS).**

Describe, compare and order objects and numbers **(see 1.4: CAPS).**

Place value: building and breaking down of numbers – use flard cards

(see 1.5: CAPS)

(3) **Problem solving**: **(see 1.6 – 1.11: CAPS)**

Same concept introduced as an interesting story.

Life world and daily experience of learners.

Practical activities that will help learners discover and explain their **own** strategy to solve problems without being told what to do.

SEAT WORK ACTIVITIES (Learners sit in mixed groups)

INDEPENDENT WORK ACTIVITIES: Give learners activities they can handle.

Individual work cards to reinforce mat activities (***Counting activities;***

Describe, Compare and order numbers; Write number symbols and number names; Complete number patterns; Decompose numbers; Identify the value of number digits; Soduko; Context free calculations; Flow diagrams; Number chains; Number pyramids; Problem solvingactivities)

OTHER PRACTICAL INDEPENDENT WORK ACTIVITIES (MIXED GROUPS)

Learners do these with given activity cards: puzzles, construction of 3D objects with straws, sort out shapes and create own patterns, measure (capacity, mass, length) and data handling activities.

MAT RESOURCES AND GROUP ROTATIONS

- Teacher's book for observation & assessment
- Mat books for each group
- Counters: beans
- Pencils
- Flard cards
- Small chalkboard
- Other concrete objects (for **practical** activity if necessary)

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OTHER FACTORS THAT CONTRIBUTE TO A SUCCESSFUL MATHEMATICS CLASSROOM

- Thorough planning and preparation
- Integration supports language development
- Use of resources
- Using cooperative learning strategies
- Creating a conducive learning environment
- Teamwork
- Modeling positive behavior and leading by example
- Creating a positive culture
- Initiating staff development activities
- Establishing good working relationships
- Setting up support structures

EXEMPLAR OF A CLASSROOM SETTING

<table>
<tr><td>5</td><td colspan="7">ORGANISATION</td></tr>
<tr><td rowspan="5">30</td><td>RED</td><td colspan="5">BLUE</td><td>GREEN AND YELLOW</td></tr>
<tr><td rowspan="4"><u>Mat activities</u>:
(i) Counting –
(a) Estimation more than 650
(b) Group counting: 5, 10, 25

(ii) Manipulating numbers:
(a) Mental Maths
(oral activity)
Up and down the number ladder: Compare numbers to 200.</td><td colspan="5"><u>Seat work activities</u>:
Number concept
(a) Write down the number names and number symbols of given number
Number sequence
(b) Copy , extend and describe simple number sequence.</td><td rowspan="4"><u>Seat work activities</u>:
Repeated addition and subtraction
(a) Number chain activity
Manipulating numbers in different ways
(b) Break down and build up given numbers
Problem solving
(c) Fikile bought 64 oranges. She puts 9 oranges in a packet, How many packets will she get?
Context free calculations
(d) Rounding off to nearest</td></tr>
<tr><td>537</td><td>542</td><td></td><td></td><td>557</td></tr>
<tr><td></td><td>642</td><td>647</td><td></td><td></td></tr>
<tr><td>737</td><td></td><td>747</td><td>75
2</td><td></td></tr>
</table>

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5 30	(b) **Physical activity**: Breaking down and building up of different numbers up to **650** using flard cards. **(iii) Problem solving:** **(oral and practical)** Multiplication problems with answers up to **75.** "I have **6** boxes of biscuits. In each box I have **12** biscuits. How many biscuits do I have altogether?" (Children do their **own** plan to solve these problem)	**Problem solving** (c) Addition and subtraction problems: Tumelo has 557 marbles and her brother Neo has 120 more. How many marbles does Neo have? (d) How many marbles are there altogether? **Context free calculations** (e) Worksheets: Addition and Subtraction sums up to **700** (f) Pyramid sums up to **500**	10 up to **400** (d) Worksheets: Multiplication sums up to 50 (e) Flow diagram sums up to **400**
	CHANGE OF GROUPS AND ACTIVITIES		
20	**Seat work activities** Same as the **blue** group work.	**Mat activities** Same as the **red** group with higher number range. Counting - **700,** mental maths - **300**, breaking down and building up numbers – **700**, problem solving – **100.**	**Practical Activities** Copy and extend geometric patterns with given activity cards. Copy and extend own patterns
	TIDYING UP AND MARKING OF SEAT WORK		

REINFORCING THE UNDERSTANDING OF PLACE VALUE THROUGH NUMBER BASES

Lerato Mathenjwa:

TEACH South Africa Ongoing Support and Training Manager

Zanele Monareng:

TEACH South Africa Ambassador teaching at Olien Comprehensive School in the Free State.

The purpose of this workshop is to give ideas on Using Number Bases to reinforce the understanding of Place Value. It is well known that in many disadvantaged schools, learners perform at a level much lower than the Grades in which they are studying. This can place a huge challenge for teachers to close existing learning gaps in learners, considering the quantity of work that has to be covered each year. If these gaps are not addressed early enough, learners progress to subsequent grades with even greater gaps in higher classes.

*Although place value is taught up to Grade 6 in the curriculum, learners in Grade 7, 8 even 9 tend to struggle with place value related concepts such as rounding off, and working with the four operations, among others. Whilst solidifying the understanding of Place value may improve the understanding of such concepts, it can also assist in the practice of exponents and finding the general term of a sequence. The activities in this workshop could be used by teachers as **expanded opportunities** for learners in the senior phase who struggle with the place value concept.*

TARGET AUDIENCE:	Senior Phase
DURATION:	One Hour
NUMBER:	Not more than 50.

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MOTIVATION FOR THIS WORKSHOP

Most learners get to Grade 7, 8 and 9 still lacking in the basics of place resulting in their inability to access higher concepts that require a deep understanding of place value. Place value is taught up to Grade 6 and by this time they should be fluent in their four operations calculations. However, this is not the case and it makes it difficult for teachers to simply teach at the level of the Grade.

If teachers were to simply give them exercises in basic concepts, learners will be bored and undermined. The activities designed here will give the learners adequate challenge yet simple concepts to practice lower concepts. The activities can be used as an intervention strategy at extra time rather than do it in class.

Teachers will benefit by receiving materials that are readily available for use.

HOW CAN BASES REINFORCE THE KNOWLEDGE OF PLACE VALUE.

Learners in senior phase may feel uncomfortable doing activities extracted purely from lower classes textbooks although lack of mastery of concepts learned at these lower classes are a huge hindrance to their progression in the learning of higher concepts. The use of activities such as the ones below affords them enough challenge for their age yet are easy enough to complete individually once the concepts have been taught to them.

Number bases

A **base** is a method of expressing numbers using place value which is the value of a digit based on its specific position. With the use of columns and a number pattern that will help in the understanding of the concept of bases can be formed.

Firstly the column should always be read from the right to left; hence the first furthest column on the right always begins with one.

Position	**6**	**5**	**4**	**3**	**2**	**1**

Starting Point

Using the base x, and raising the base x exponentially (these bases represent place values), the base at position 1 always starts with x^0 and exponents continue consecutively as the position increase consecutively simultaneously. Reading the column from right to left we get:

←

Position	6	5	4	3	2	1
Bases	x^5	x^4	x^3	x^2	x^1	$x^0 = 1$

Starting Point

Applying the above to a number, for example the number 123 in other words in base x, 123 means:

We write the number backwards from right to left starting from position 1.

Position	6	5	4	3	2	1
Bases	x^5	x^4	x^3	x^2	x^1	$x^0 = 1$
Number				1	2	3

Determining base x of 123 means:

$$123_x = (1 \times x^2) + (2 \times x^1) + (3 \times x^0)$$

Position	6	5	4	3	2	1
Bases	$2^5 = 32$	$2^4 = 16$	$2^3 = 8$	$2^2 = 4$	$2^1 = 2$	$2^0 = 1$
Number				1	2	3

Using the same number with base 2 (123 base 2):

$$\therefore 123_2 = (1 \times 2^2) + (2 \times 2^1) + (3 \times 2^0)$$

$$= 4 \quad + \quad 4 \quad + \quad 3$$

$$= 11$$

Base Five and Base Two numeration systems

In the decimal system, numbers are grouped in 10's whereas in base five they are grouped in 5's and grouped in 2's in base two. The binary system is mostly used in computer systems where the digits zero and one operate as "on (1)" and "off (0)" in the case of switches in simple terms.

Example: 28 in base 1 0 is 2 tens and 1 eight but in base five there are 5 fives

28_{10} is:

TENS	UNITS
	
2	**8**

Fig 1 A representation of 28 in base 10

103_5 is:

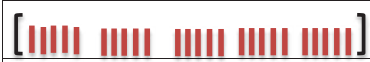
TWENTY-FIVES	FIVES	UNITS
		
1	**0**	**3**

Fig 2 A representation of 28 in base 5

When you say the number, it reads, one, zero, three base five and **NOT** one hundred and three base five. It is important to emphasise that in base 10, 28 has TWO place values whereas it has THREE in base five. The two numbers represent 28 one in base 10 and the other in base 5. Every number in a particular base has all its digits less than the base. For example, the largest digit in base 5 is 4 and the largest digit in base 8 is 7.

Mendieta states that "The depth of conceptual understanding one has about a particular mathematical concept is directly proportional to one's ability to <u>translate</u> and <u>transform</u> the <u>representations</u> of the concept across and within a wide variety of <u>representational systems</u>." It is highly possible that with this in mind, the use of knowledge of bases will increase the learners' understanding of place value as they represent numbers across different number bases.

Base Five and Base Two numeration systems

Activity 1: Make a representation of 28 for Base Two as shown above and write the number of how many place.
[5 min]
Place Values in other Bases:

Table 1: Converting 28 to base 2 and 5

The Number 28 in base 10 converted to base 5 and 2							
Base	**Number**	**Place Value**					**No. of Places**
Base 10	**28**				10^1	10^0	**2**
					10	1	
					$10 \times \mathbf{2}$	$1 \times \mathbf{8}$	
					$\mathbf{20 + 8 = 28}$		
Base 5	$\mathbf{103_5}$			5^2	5^1	5^0	**3**
				25	5	1	
				$25 \times \mathbf{1}$	$5 \times \mathbf{0}$	$1 \times \mathbf{3}$	
				$\mathbf{25 + 0 + 3 = 103_5}$			
Base 2	$\mathbf{11100_2}$	2^4	2^3	2^2	2^1	2^0	**5**
		16	8	4	2	1	
		$16 \times \mathbf{1}$	$8 \times \mathbf{1}$	$4 \times \mathbf{1}$	$2 \times \mathbf{0}$	$1 \times \mathbf{0}$	
		$\mathbf{16 + 8 + 4 + 0 + 0 = 11100_2}$					

The table above shows how to convert base 5 and 2 to base 10.
Another method to convert a number from base 10 to another base is shown below. This method can be introduced much later when the concept of place value is well grasped:

Table 2: Another method of converting 28 to base 2 and 5

Converting 28 base 10 to base 5 and 2			
28 in base 5		5	28
	(28÷5)	5	5 rem **3**
	(5 ÷ 5)	5	**1 rem 0**
	(1 ÷ 5)	5	**0 rem 1**
In reverse order the answer is $\mathbf{103_5}$			
28 in base 2		2	28
	(28÷2)	2	14 rem **0**
	(14÷2)	2	7 rem **0**
	(7 ÷ 2)	2	3 rem **1**
	(3 ÷ 2)	2	1 rem **1**
	(1 ÷ 2)	2	**0 rem 1**
In reverse order the answer is $\mathbf{11100_2}$			

READING IN REVERSE ORDER

Converting other number bases to base 10.

Activity 2: Convert the following numbers (in base 10) below, to the base shown in the bracket.

a) 103 (base 5)
b) 76 (base 2)
c) 46 (base 3)
d) 245 (base 8)
e) 105 (base 4)

[5 min]

The exercise below is for learners to convert numbers from other bases to base 10: It is encouraged that they always write the place values in exponential form and then as a value.

Activity 3: Write the following numbers in base 10.

a) 1110011_2
b) 4031_5
c) 705_8
d) 3230_4
e) 1221_3

[5 min]

Addition and Subtraction of number bases other than 10

Activity 4: Subtract or add the following numbers.

a) $1110011_2 \quad - \quad 1101110_2$

b) $4031_5 \quad + \quad 4114_5$

c) $705_8 \quad - \quad 656_8$

d) $3230_4 \quad + \quad 3333_4$

e) $1221_3 \quad - \quad 1122_3$

[5 min]

Examples:

Table 3: Illustration of addition or subtraction of two numbers with other bases

	64	32	16	8	4	2	1
a)	1	1	$^{0}\cancel{1}$	$^{1}\cancel{0}$	$^{2}\cancel{0}$	1	1
	1	1	0	1	1	1	0
	0	**0**	**0**	**0**	**1**	**0**	**1**
Answer							**101_2**
b)			625	125	25	5	1
			^{1}	4	$^{1}\cancel{0}$	$^{1}3$	1
				4	1	1	4
			1	**3**	**2**	**0**	**0**
							13200_5
CHECKING THE ANSWERS BY CONVERTING ALL NUMBERS TO BASE 10							
a)	64	32	16	8	4	2	1
	1	1	1	0	0	1	1
							64 + 32 + 16 + 0 + 0 + 2 + 1 = 115
	64	32	16	8	4	2	1
	1	1	0	1	1	1	0
							64 + 32 + 0 + 8 + 4 + 2 + 0 = 110
							$115 - 110 = 5$ And 5 in base 2 is **101_2**
b)				125	25	5	1
				4	0	3	1
				500 + 0 + 15 + 1 = 516			
				125	25	5	1
				4	1	1	4
							500 + 25 + 5 + 4 = 534
	$516 + 534 = 1050$ And 1050 in base 5 is **13200_5**						

Prime Numbers and Number bases:

A **prime number** is a number that is divisible by only two numbers, one and itself. Prime numbers are also an important concept and when well founded and understood learners will not experience problems with factorising. Upon conducting an investigation, Ferguson found an interesting pattern with prime numbers. The table below shows 36 counting numbers arranged in rows of 6. The first column and fifth have most of the numbers as prime numbers except 2 and 3 which are not in those column.

Base Ten					
1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36
37	38	39	40	41	42
43	44	45	46	47	48
49	50	51	52	53	54
55	56	57	58	59	60

Table 4: Counting Numbers arranged in 6 rows

When the same numbers are written in base 6, the pattern is more distinct where the first and fifth column have a general form $6n \pm 1$.

$7, 13, 19, \ldots 6n + 1$

$5, 11, 17, \ldots 6n - 1$

Base Six					
1	2	3	4	5	10
11	12	13	14	15	20
21	22	23	24	25	30
31	32	33	34	35	40
41	42	43	44	45	50
51	52	53	54	55	100
101	102	103	104	105	110
111	112	113	114	115	120
121	122	123	124	125	130
131	132	133	134	135	136

Table 5: Numbers in base 6 arranged in 6 rows

Learners can work on patterns in a similar way.

Activity 5:

a) Arrange counting numbers in rows of 4 up to 40. Mark all the prime numbers. Which rows have the most prime numbers?
b) Change the numbers above to base four.
c) Find the general form of the sequences with most prime numbers.

[5 min]

Further:

Activity 6:

a) Arrange counting numbers in rows of 12 up to 60. Mark all the prime numbers. Which rows have the most prime numbers?
b) Change the numbers above to base twelve.
c) Find the general form of the sequences with most prime numbers.

[5 min]

In conclusion, learners who have missed the grasp of key basic concepts need to catch-up in order to progress faster with higher concepts. Activities given above can be added as intervention for the catch-up.

REFERENCES

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Aneenberg, A. Learner: Learning Math http://learner.org/courses/learningmath/number/session3/part_a/index.html

Mendieta, G. Pictorial Mathematics: http://books.google.co.za/books/about/Pictorial_Mathematics.html?id=vmBozj50AtwC&redir_esc=y

ROCKET CAR-MATICS

Christopher Maxwell, Dave Rowley
Edit Microsystems, Durban

TARGET AUDIENCE:	Foundation ; Intermediate ; Senior Phase and FET teachers
DURATION:	1 hour
MAXIMUM NUMBER OF PARTICIPANTS:	32

The Bloodhound SSC (Super Sonic Car) project to break the world land-speed record is taking science and mathematics forward to inspire a generation to think differently and scientifically about the world we live in. Using the mathematics of the Bloodhound SSC to take it to a speed of 1610 km/h, this workshop will explore the shapes and equations to give a successful mathematical solution to a stable system.

Build a Rocket Car using mathematics or simply 'Rocket Car-matics'. This workshop takes snippets of the mathematics curriculum from each stage ranging from Foundation to FET level and combines them to formulate a 'Rocket Car' system which can speed or sink its way to a finish line. Working in groups of 4 learners per group it provides an applicable solution using symmetry and geometry, percentage and proportion, graph sketching and interpretation, as well as stability and matrix equations. Come put your mathematics to a good test and build a stable system with 'Rocket Car-matics'.

MOTIVATION FOR THIS WORKSHOP

This workshop will bring a development edge where younger learners can see their mathematics in a practical example. Introducing one key part of the mathematics curriculum for each phase in a group environment. It emphasizes the importance of mathematics in a real-world example, the Bloodhound SSC project.

INTRODUCTION TO WORKSHOP

Rocket Car-matics has been designed using STEM product aids. The Bloodhound SSC is a car that has been designed to travel faster than the speed of sound as well as break the world land-speed record. The relevance is that the car will be run exclusively in South Africa and the driver of the car, Andy Green, apart from being an RAF fighter pilot also has a first-class pass in mathematics from Oxford University, UK, and is a huge ambassador for mathematics. The project is driven by a worldwide education program to inspire a generation and bring mathematics alive. This workshop looks at the mathematical fronts of the car for a range of levels. It has worksheets that are used to display the skills as well as using mathematical software on an interactive 'Smart Board'.

DESCRIPTION OF CONTENT:

There will be 8 groups of 4 people per group in 4 categories. Each category represents a specific education phase. The groups are: foundation, intermediate, senior and FET.

Each age group will develop the mathematics of a specific area of the car. The main topics covered by the stages will be:

Foundation: Recognition of geometrical shapes and combining them to build one large shape.

Intermediate: Proportions and Percentages and ratios such as using fractions and decimals.

Senior: Interpretation of graphs, students will be looking at number lines and interpreting them with a graph's of velocity and acceleration. Also some graph sketching is involved in this level.

FET: Solving differential equations to calculate. Solving stability equations and matrix equations of the rocket car system.

Each phase will have time to plan in their group which strategy to follow in order to solve their particular problem. After planning the group will have time to put their work up on a Smart Board which will be provided by Edit Microsystems. The Smart board will be visible to all and will showcase the problem solving taking place quite clearly to others and audience.

The workshop has 2 different parts, each part differing from the previous as well as testing more of the groups' ability. Each part will take a total of 30 minutes.

The activities and worksheets to be used in the workshop:

The worksheets are being developed as the workshop will be undergoing its pilot run in the final week of April. Once they are finalised they will sent through to AMESA. The basic outline will be the following:

Foundation: Identifying and using shapes with different orientations.

Intermediate: A set of ratios and percentages of the 'Rocket Car' used to calculate how the car can be put together to minimize size.

Senior: An equation that will represent a part of the car that needs a curve fitted. They will be using curve sketching methods and derivatives to give a set of results that will represent the car graphically.

FET: Worksheet includes minimizing equations, derivatives of equations and using Lyapunov functions to calculate regions of stability. Introduction to Navier-Stokes equations which is used in CFD (Computational Fluid Dynamics) of the Bloodhound SSC project.

DERIVATIVE FUNCTION USING GEOGEBRA

Ingrid Mostert

Kelello Consulting

Developing an intuitive understanding of the derivative function is an important part of teaching calculus. In this workshop participants will use a dynamic GeoGebra worksheet to explore an intuitive concept of the derivative of a function by considering many different functions. The workshop will cater for participants who have never used GeoGebra as well as those who are comfortable using it.

TARGET AUDIENCE:	FET Phase
DURATION:	1 hour
MAXIMUM NO. OF PARTICIPANTS:	30

MOTIVATION FOR THIS WORKSHOP

Many learners never develop a conceptual understanding of what a derivative function is. For this concept to develop learners must first have an intuitive understanding of the instantaneous rate of change at a point on a function before exploring the idea that the set of all these instantaneous rates of change form a new function.

This workshop presents participants with an easy way of helping learners develop such an intuitive understanding of the derivative function. The dynamic GeoGebra worksheet can be adapted for use in each participant's class.

DESCRIPTION OF CONTENT OF WORKSHOP

In the first part of the workshop (15min) participants will be given a function and will draw in the gradient at 8 points on the function after which they will determine the gradient of the function at each of these points. They will then determine the equation of the function going through the 8 points with x-coordinate the same as the x-coordinate of the original 8 points and y-coordinate being the gradient of the tangent to the function at that point.

Participants will then (30min) use a GeoGebra dynamic worksheet to go through the same process but much more quickly. Because using GeoGebra makes it possible to estimate the derivative function more quickly, it is possible to explore a variety of functions including quadratic functions, cubic functions, linear functions, exponential functions, logarithmic functions and trigonometric functions.

Next (15min) will use cards which need to be matched to determine the derivative rules. They will have the opportunity to discuss how useful they think such an activity would be in their classes.

ACTIVITIES AND WORKSHEETS TO BE USED EXPLORING INSTANTANEOUS RATES OF CHANGE AT POINTS ON A FUNCTION

Work with a partner:

Activity 1

1. On first show board, draw the function of $x^2 - 4$.
2. Choose 8 points on the function and for each point:
    a) Draw in the tangent to the function at the point as accurately as possible.
    b) Work out the gradient of the tangent at that point by reading off values on your axes.
    c) On the second show board, plot the gradient of the tangent vs the x-coordinate for each of the 8 points.
    d) What do you notice?
    e) Determine the equation for the function on the second show board by using your knowledge of the equation of a straight line.
    f) The new function is called the derivative function and instead of an input of x and an output of y we now have an input of x and an output of…

Activity 2

1. Open “Exploring Instant Rate of Change” folder.
2. Open “Quadratic” GeoGebra worksheet.
3. The orange point lets you fix the gradient of the line passing through the point with same x-coordinate on the function. Move the orange points so that line becomes the tangent to the function at that point.
4. What do you notice about the orange points?
5. Use the other GeoGebra worksheets to complete the table on the next page.

Function	**General Formula**	**Function through Orange Points**
Quadratic Function		
Cubic Function		
4^{th} degree polynomial		
Exponential Function		
Logarithmic Function		
Sin Function		
Cos Function		
Tan Function		
Straight Line		

We can use this method to determine the equation of the derivative function for many functions but it is long and tedious.

Activity 3

1. In your pair match the cards according to the types of functions that they have.
2. To do this fill in the type of function in the space provided on each card.
3. Start with one pile and try to formulate a short cut rule you can use to get from the function to its derivative function.
4. Use the GeoGebra worksheets to test your rules:
    a) Choose a type of function
    b) Write down an example of this type of function
    c) Write down what TYPE of function the derivative function will be
    d) Use your rules to work out the equation for the derivative function
    e) Draw the function in the appropriate GeoGebra worksheet
    f) Click on "Show Derivative" to check whether your rule worked

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SPACE, SHAPE AND MEASUREMENT: INTEGRATING LESSON CONCEPTS AND EXPERIENCES

Tatiana Sango

Teaching Mathematics to Additional Language Learners: Pearson Teacher-training course

This workshop provides instruction in the following areas: Measurement: Meaningful activities; Area and perimeter, polygons: Building on prior knowledge; Surface area and volume of 3D objects: Integrating language skills.

In Part 1 of the workshop, participants learn techniques to teach mathematical concepts to increase learner understanding and engagement. In Part 2, we will demonstrate how to use the techniques with learners in the classroom. Participants take the role of learners so that they experience and analyse the activity. In Part 3, the participants learn how to teach academic vocabulary that learners need in order to be successful. The workshop is designed to deepen and expand participants' knowledge of teaching techniques and classroom strategies.

MEASUREMENT: MEANINGFUL ACTIVITIES

Lesson Outcomes

To learn about:

- Meaningful activities that are relevant to content and language objectives.
- How to assist learners to connect the known to the unknown.
- How to make activities relevant to learners' lives.
- Approaches to teaching measurement.
- Providing opportunities for learners to interact with words in a variety of ways.

AREA AND PERIMETER, POLYGONS: BUILDING ON PRIOR KNOWLEDGE

Lesson Outcomes

To learn about:

- Ways of building on learners' prior knowledge when introducing new content and new concepts.
- Ways of connecting content to real-life experiences and learners' personal experiences.

- Approaches to teaching perimeter and area through practical means.
- Providing learners with opportunities describing vocabulary using different techniques.

SURFACE AREA AND VOLUME OF 3D OBJECTS: INTEGRATING LANGUAGE SKILLS

Lesson Outcomes:

To learn about:

- Meaningful activities that integrate lesson concepts with language practice.
- Approaches to teaching surface area and volume of three-dimensional (3D) objects through practical means.
- Providing learners with opportunities to describe vocabulary using different techniques.

SUGGESTED WORKSHOP ACTIVITY

1. **MATHEMATICAL Activity 1 (MENTAL MATHS): PICK A BOX**

Activity objective	To determine the volume of various 3D objects
Approximate time	10 minutes
Materials	Projector/TV PowerPoint Slide 6 Effective mathematical activities (III) (Handout 18, Workbook p. 23) Six boxes, numbered 1–6, of which the actual volume has been calculated in advance

Procedure:

Note: The Facilitator is expected to model classroom teaching and will take the role of the teacher for these sample activities. Participants will take the role of the learners. Make it clear to participants that the pace used in this presentation is much faster than the pace they would need to use with their learners in the classroom.

a) Class Greet the participants as if they were learners in a Mathematics class. Explain that they will take part in three sample mathematical activities as learners of mathematics. Tell the class that this lesson is about determining the volume of various 3D objects. Allow 5–8 minutes for the actual activity (just enough until participants get a feel for the activity).

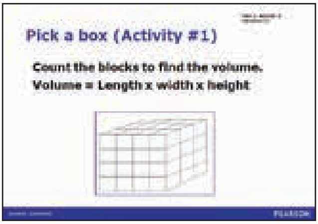
b) Facilitate Activity #1. Follow the step-by-step instructions below on how to facilitate this activity.

Activity1: Step-by-step instructions

1. Ask participants for the definition of volume. (The amount of space occupied by a 3D object)

2. Remind participants that there are two ways in which they can find the volume of a rectangular box or an object made up of rectangular boxes. (Count the blocks or use the formula V = length x width x height.)

3. Show PowerPoint Slide 6.

PowerPoint Slide 6	

4. Discuss how to use these methods to arrive at the answer. (Counting: Each layer has 4 x 3 = 12 blocks. There are 4 layers, so there are 48 blocks. Calculating using a formula: V = 3 x 4 x 4 = 48 blocks.)

5. Remind participants that if the length of each side of the cube is 1 cm, its volume is 1 cubic centimetre.

6. **Groups** Organise participants into six groups.

7. Give each group a box.

8. Draw the following table on the board:

	Estimated volume					
	Box 1	Box 2	Box 3	Box 4	Box 5	Box 6
Group 1						
Group 2						
Group 3						
Group 4						
Group 5						
Group 6						
Actual volume						

9. Explain that each group should look at their box and estimate the lengths of the sides, and work out an estimate of the volume of the box in cubic centimetres. This should be done without doing any actual measurements.

10. Ask a representative of each group to write down their answers in the table on the board.

11. After each group has written their estimates, fill in the actual volumes of the boxes (in the bottom row of the table).

12. Participants should decide which group's estimate was closest to the actual volume of each box.

c) Close the Mathematics class simulation. Refer participants to Handout 18, Effective mathematical activities (III). Ask participants if the activity 'Pick a box' applied the content and language knowledge and integrated all language skills.

Teaching points

- By introducing an element of competition here, the activity provides some fun, which helps to make the mathematical concept more inviting.
- Cardboard boxes of all sizes are readily available at no cost. This makes them ideal manipulatives.
- The hands-on use of boxes gives learners an opportunity to engage with three-dimensional properties, and to think about length, width and height and how they are combined to calculate volume. It is essential for a full understanding of the difference between length, area and volume that learners have this opportunity.

DEMYSTIFYING PYTHAGORAS

Ian Schleckter

Free State Department of Education, Motheo District Office.

This workshop is aimed at Senior Phase teachers as audience in discovering a different approach in teaching the theorem of Pythagoras.

The content introduces us to the historical environment of Pythagoras, integrating history and geography with mathematical theory, leading to mathematical practice and challenging problem-solving strategies, including measurement, properties of triangles, squares and rectangles, solving equations, approximating answers using an approximation of the square root. Real-life examples will be identified and treated.

Teachers will engage in stunning and fun-filled practical activities that can be brought into the classroom, allowing for scaffolding skills and ultimately utilizing the theorem in practical mathematical problems.

TARGET AUDIENCE:	Senior Phase teachers
DURATION OF WORKSHOP:	1 hour
MAXIMUM # OF PARTICIPANTS:	40

MOTIVATION FOR THIS WORKSHOP

The theorem of Pythagoras (and its applications), are rich in many aspects of mathematics that confront learners at school. Aspects such as properties of triangles, area, height, measurement, exponents, squares, problem-solving, trigonometry, geometry, etc., are addressed when teaching / dealing with the theorem of Pythagoras. It integrates various topics in mathematics. Unfortunately, learners fail to achieve satisfactory results in problems where the theorem of Pythagoras is applied. Teachers tend to "dump" the theorem of Pythagoras without taking into account the essential pre-knowledge needed or they fail to create a solid understanding of why the theorem is applied, integrating geometry, algebra, trigonometry, etc.

DESCRIPTION OF CONTENT

Using a PowerPoint presentation to provide context (historical), and to introduce the content by using animation to outline and emphasise different aspects of the theorem, teachers will be led to understand the scaffolding process before the actual theorem is introduced. Basic pre-knowledge is dealt with, followed by the unfolding of the theorem before the application of the theorem is revealed.

Teachers will engage in paper activities, unfolding the mysteries of the theorem, even do singing (rap).

ENGAGING WITH NOKIA MOBILE MATHEMATICS – A LOCAL ONLINE FET MATHS SERVICE WITH NO SUBSCRIPTION FEES

Garth Spencer-Smith

Kelello Consulting (associate) and Ukufunda Consulting

Nokia Corporation, in partnership with the national Department of Science and Technology, has developed a free, online, CAPS-aligned mobile learning mathematics service for Grade 10-12 learners in South Africa, called the Nokia Mobile Mathematics service (http://momaths.nokia.com/za/m/). The service is intended to be used primarily after school hours as a homework administration and revision resource. Research on its impact on mathematics attainment over one academic year has been conducted in 30 schools using school assessment data (Roberts & Spencer-Smith 2014). Further research on impact utilising standardised assessments is currently underway.

MOTIVATION FOR THIS WORKSHOP

Nokia Mobile Mathematics is a free, online, fully CAPS-aligned service that learners can access 24/7/365 in order to improve their Maths performance. It will help participants in that they will learn the benefits of the service for their learners; have an opportunity to 'play' on the service; and learn what data they can get from the service about their learners' usage and performance.

TARGET AUDIENCE: FET

DURATION: 1 hour

MAXIMUM NUMBER OF PARTICIPANTS: The number of participants is limited only by the number of computers that are available for their use (it will need to be held in a computer lab, with internet access, with one teacher per computer)

DESCRIPTION OF CONTENT OF WORKSHOP:

30 minutes: description of the service by means of a PowerPoint presentation

30 minutes: teachers will access the service and 'play' on it themselves

The activities and worksheets to be used in the workshop (maximum 8 pages): There will be a hand-out providing details of the service (e.g. what it is; its benefits; and how to access it). This is a colour brochure which is still in development, and will be available by the congress.

THIS WORKSHOP WILL ENSURE YOU LEARN HOW THIS SERVICE:

- Will benefit your learners (e.g. providing access to over 9 000 questions; divided up by grade, topic and difficulty level)

- Will benefit you as an FET Maths teacher (e.g. information on the uptake, use and performance of your learners on the service)

About half of the workshop will be spent on accessing the service yourselves and seeing exactly what it has to offer. Computers will be available however you can also make use of your personal mobile devise should you have internet connectivity.

REFERENCES

Roberts, N & Spencer-Smith G, et al., (2014) *From Challenging Assumptions To Measuring Impact Of The Nokia Mobile Mathematics Service In South Africa*, SABEC abstract presentation, SA Basic Education Conference, 31st March – 1st April, 2014, Caesars Palace, Johannesburg (Full paper in press))

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THE COUNTING PRINCIPLE

Desiree Timmet
Statistics South Africa

TARGET AUDIENCE: Further Education and Training educators
DURATION: 1 hour
MAXIMUM NUMBER OF PARTICIPANTS: 30

MOTIVATION FOR THIS WORKSHOP

Probability is a relatively new topic in School mathematics and was also not part of a considerable number of teachers' pre- service training and studies. Teaching Data handling and Probability, poses a real challenge to some educators because their knowledge in this content area is limited. In this workshop we will focus on The Counting Principle, a topic in the grade 12 curriculum. The following key concepts are covered in this topic (as indicated in the Curriculum Statement).

- *Probability problems using Venn diagrams, tree diagrams, two way contingency tables and other techniques (like the fundamental counting principle) to solve probability problems (where events are not necessarily independent).*
- *Apply the fundamental counting principle to solve probability problems*

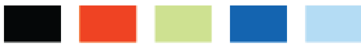
COUNTING PRINCIPLES

Introductory example: Suppose you are buying a new car.

There are **2** body styles:

sedan or hatchback

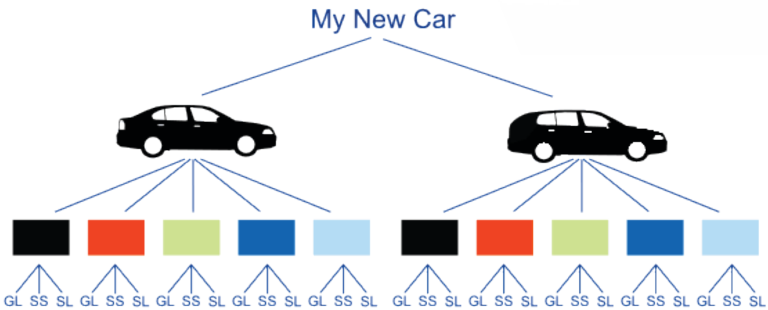
There are **5** colours available:



There are **3** models:

- GL (standard model),
- SS (sports model with bigger engine)
- SL (luxury model with leather seats)

How many total choices?



It is possible to list all possible outcomes using a tree diagram. When you have many possible outcomes, a tree diagram can become very messy and it becomes difficult to count the possible outcomes.

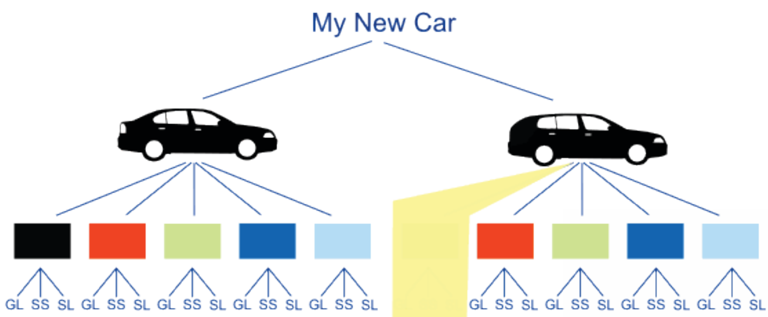
Counting principles help you to count the possible outcomes without drawing a tree diagram.

Independent or Dependent?

If one choice affects another choice (i.e. **depends** on another choice), then a simple multiplication is not right.

You are buying a new car ... but ...

The salesman says **"You can't choose black for the hatchback"** ... well then things change!



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You now have only 27 choices.

Because your choices are **not independent** of each other.

But you can still make your life easier with this calculation:

$$\text{Choices} = 5\times3 + 4\times3 = 15 + 12 = 27$$

Arrangements with repeats

It will assist in understanding the counting principles by thinking back to the tree diagrams that you used in previous grades.

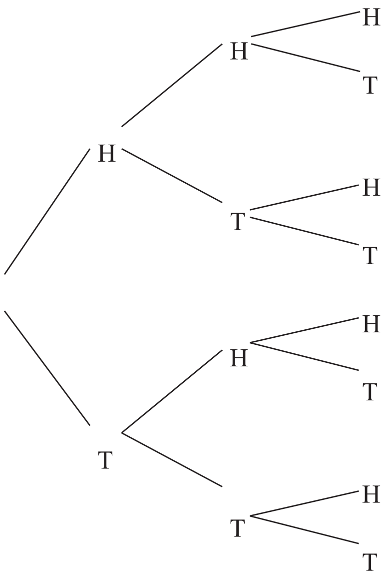
Example 1

a) A coin is tossed twice. How many outcomes are there?
b) A coin is tossed three times. How many outcomes are there?
c) A coin is tossed four times. How many outcomes are there?
d) A coin is tossed twenty times. How many outcomes are there?

Solution

a)

b) 1st toss 2nd toss 3rd toss Possible outcomes



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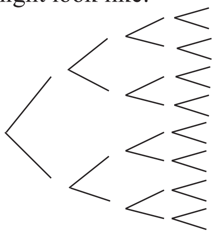
There are $2 \times 2 \times 2 = 8$ outcomes in total.

c) It becomes more difficult to draw a tree diagram to show all the possible outcomes when a coin is tossed 4 times. Below is a simple sketch of what one might look like:



When a coin is tossed four times:
There are two possible outcomes for the first toss.
There are two possible outcomes for the second toss.
There are two possible outcomes for the third toss.
There are two possible outcomes for the fourth toss.
There are $2 \times 2 \times 2 \times 2 = 16$ outcomes in total.

d) It is not easy to draw a tree diagram for 20 tosses, but you should be seeing a pattern.
If a coin is tossed 20 times, there will be $2 \times 2 \times 2$…. (to 20 terms)
Number of possible outcomes = ……………………………….

- The ***fundamental counting principle*** is a quick method for calculating numbers of outcomes using multiplication.

- The fundamental counting principle states:

  Suppose there are n_1 ways to make a choice, and for each of these there are n_2 ways to make a second choice, and for each of these there are n_3 ways to make a third choice, and so on.
  The product $n_1 \times n_2 \times n_3 \times \ldots \times n_k$ is the number of possible outcomes.

In simple language the fundamental counting principle says:
"If you have several stages of an event, each with a different number of outcomes, then you can find the TOTAL number of outcomes by multiplying the number of outcomes of each stage."

Arrangements without Repeats

Sometimes we have examples where an event can only be used once.

Activity 1

a) Two counters marked A and B are randomly drawn from a box. When a counter is taken, it is not returned. How many ways can these letters be drawn, i.e. how many possible outcomes are there?

b) Three counters marked A, B and C are randomly drawn from a box. When a counter is taken, it is not returned. How many possible outcomes are there?

Factorial Notation

- The arrangement of numbers $4 \times 3 \times 2 \times 1$ can be written as 4!
  - You say '***4 factorial***'.
  - $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1$
  - $n! = n \times (n-1) \times (n-2) \times (n-3) \times (n-4) \times \ldots \times 4 \times 3 \times 2 \times 1$
- Factorial notation is used for finding the total number of outcomes without repeats.
- Most scientific calculators have a factorial key.

On the Casio fx-82ZA PLUS, the factorial key ($x!$) is next to the (x^{-1}).
To calculate 6!, enter the number (e.g. 6) then press [SHIFT] [x^{-1}] (x!) [=]

On the Sharp EL-W535HT, the factorial key ($n!$) is next to the 1.
To calculate 6!, enter the number (e.g. 6) then press: [2nd F] [4] (n!) [=]

Activity 2

a) A three digit code is made up of numbers 3, 5 and 7. The digits may be repeated. How many different codes are possible?

b) A three digit code is made up of numbers 3, 5 and 7. Each digit is used only once. How many different codes are possible?

- WHEN REPEATS ***ARE ALLOWED***, *you find the total number of outcomes by multiplying the number of possible outcomes in each stage of an event.*

- WHEN REPEATS ***ARE NOT ALLOWED***, *you find the total number of outcomes by multiplying the number of possible outcomes* ***that are left*** *in each stage of an event.*

- Sometimes you don't want to arrange all the items, but only some of them.

Activity 3

1) Write each of these as a product of digits (e.g. $3! = 3 \times 2 \times 1$)
   a) 4!
   b) 7!

2) Use your calculator to determine each of the following (e.g. $5! = 120$)
   a) 6!
   b) 8!

3) The following menu is offered at a restaurant:

Starter	Main	Dessert
Chicken livers	Peri-peri chicken	Fruit salad and ice-cream
Tomato soup	Lamb chops	Chocolate pudding
Salad	Beef rump	
	Fish and chips	

   If one starter, one main and one dessert is selected from the menu. How many combinations of starter, main and dessert could be chosen?

4) Given the numbers: 0; 1; 2; 3; 4 and 5
   a) How many 4 digit numbers can be formed if the first digit may not be 0 and the numbers may not be repeated?
   b) How many of these numbers will be divisible by 5?

5) The soccer coach needs the goal posts to be moved on the field. He randomly chooses five boys out of a group of twenty to help him. How many different groups of 5 boys can be selected?

Special Conditions

Sometimes when we are counting the number of arrangements, we are given special conditions, for example similar groups must be arranged together, or two or more elements must be put together in the arrangement.

Activity 4

A photograph needs to be taken of the Representative Council of Learners (RCL) at a school. There are three girls and two boys in the RCL and all of them need to sit in one row for the photograph.

moooi.com

a) Suppose there is no restriction on the order in which the RCL sits. In how many ways can the RCL be arranged in a row?
b) Suppose the President and the Vice-President of the RCL must be seated next to each other, in how many different ways can the RCL be arranged in a row?
c) Suppose all the girls must sit next to each other, and all the boys must sit next to each other. In how many different ways can the RCL be arranged in a row?

Identical Items in a list

Consider how many arrangements of the letters there are in the word LEEK. Here is a list of some of the possible arrangements:

LEEK	LEKE	LEEK	LEKE	
LKEE	LKEE	ELEK	EELK	Etc

- Because the letter E is repeated, we *cannot* say that there are 4! different arrangements. In fact, because 2 letters are repeated, there is half the number of different arrangements than there would be if all four letters were different.

We say that

- If there are n different items that are all different, then there are $n \times (n-1) \times (n-2)$ ... n terms or $n!$ arrangements.

- If there are n different items, but ***one item is repeated twice***, then there are
  $n \times (n - 1) \times (n - 2)$ ... n terms **divided by** 2 or 2! arrangements.
- If there are n different items, but ***one item is repeated three times***, then there are $n \times (n - 1) \times (n - 2)$ ... n terms **divided by** $3 \times 2 \times 1$ or 3! arrangements.

Activity 5

a) In how many ways can you arrange the letters in the word MEDIAN?...
b) In how many ways can you arrange the letters in the word DATA? ……………………………………………….
c) In how many ways can you arrange letters in the word PERCENTILE? ……………………………………
d) In how many ways can you arrange the letters in the term CENSUS@SCHOOL? ……………………………………

Using Counting Principles to Find Probability

- You can use these counting principles to find the number of possible outcomes, and you can also use them to find the number of favourable outcomes.

- When you know the number of possible outcomes and the number of favourable outcomes, you can work out the probability of the favourable event using the formula

$$\text{Probability of a favourable event} = \frac{\text{Number of favourable outcomes}}{\text{Number of possible outcomes}}$$

Activity 6

1) Suppose a four-digit number is formed by randomly selecting four digits without repetition from 1; 2; 3; 4; 5; 6; 7 and 8.

   What is the probability that the number formed lies between 4 000 and 5 000?

2) What is the probability that a random arrangement of the letters in the name 'PHILLIPINE', start and end in 'L'?

REFERENCES

Statistics South Africa (2013) (unpublished). Data Handling & Probability, Grades 10 -12

Department of Basic Education. (2011). *Curriculum and Assessment Policy Statement, Grade 10 – 12, Mathematics.*

Website: www.mathsisfun.com

ORGANISING AND SUMMARISING DATA

Mukhaninga Tshililo
Statistics South Africa Limpopo province

TARGET AUDIENCE: GET
DURATION: 1 hour
MAXIMUM NUMBER OF PARTICIPANTS: 30

MOTIVATION FOR THIS WORKSHOP

Data handling is still a challenge to teachers as most of them were not taught this section whilst doing their teacher training studies. I believe that this workshop will help the participants to become more familiar with ways in which data can be organised and summarised making it more meaningful for them and to enable them to teach learners with confidence.

Thus, in this workshop participants will be introduced to the meaning of data, types of data and how to organize and summarise data in different ways.

CONTENT OF THE WORKSHOP

The following activities will be covered in this workshop:

- *Recognising continuous and discrete data*
- *Organise and record data using tallies*
- *Draw up a stem and leaf display*
- *Draw up a frequency table.*
- *Arrange data into a grouped frequency table*

ORGANIZING DATA

When you first look at numerical data, all you may see is a jumble of information. You need to sort or summarise the data and record it in a way that puts order into it so that it makes more sense. One of the most common ways of sorting data is by making a list. When you make a list you write things underneath each other. Data is easy to sort into lists that are either numerical or alphabetical.

Given below are the heights of Grade 8 boys recorded by census@school 2009 in an Eastern Cape school measured in centimetres

165	148	158	150	160	165	150	156	155	164	162	160	158	148	158
140	146	160	148	152	139	165	148	160	156	158	170	155	160	148
155	158	179	170	158	161	155	160	163	178	138	172	170	156	160
160	171	140	160	170	175	148	170	177	155	167	154	160	170	155
136	179	150	167	148	160	164	167	157	165	163	140	162	178	160
170	163	162	165	175	165	152	147	180	148	170	165	167	165	165

Activity1

a) Write a list of boy's height in ascending order?

b) Which height is the most common?

Tally Table

A tally table is a mark which shows how often something happens. For each score a vertical stroke is entered on the appropriate row with a diagonal stroke being used to complete each groups of five strokes

(c) Complete a tally table for the above data

Frequency Tables

Frequency is how often something happens. The tally count for each outcome is called the frequency of the outcome. For example the frequency of the outcome 140 in no 1 above is 3

(d) Complete a frequency table for the above data

Activity 2

a) Conduct a simple survey of the members in your group to ask about the month of their birthday.

b) Record the information in a frequency table like the one below:

Month	tally	frequency
Jan		
Feb		
March		

c) In which month do most birthdays occur?

d) In which month do the least birthdays occur?

Stem and leaf displays

Tally tables become uncomfortably long if the range of possible values is very large, as with the frequency tables above. John Tukey, an American mathematician and statistician, devised a way of organising data in the 1960s called a **stem and leaf display,** convenient alternative in which the stem represents the most significant digit.

- Stem-and-leaf displays can be used both with discrete data and with continuous data (rounded off to the nearest whole number).
- Stem-and-leaf displays retain the original data information, but present it in a compact and more easily understandable way – an efficient data summary.

For example, suppose the members of your class scored the following marks in a mathematics test:

32	56	45	78	77	59	65	54
54	39	45	44	52	47	50	52
40	69	72	36	57	55	47	33
39	66	61	48	45	53	57	56
55	71	63	62	65	58	55	51

This list of numbers makes no sense as it is. You can organise the data by using a stem and leaf display where the **tens digits** form the **stem** and the **units digits** the **leaves**

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To draw the display, write all the **tens digits** in the **left hand column** of a table. The **unit's digits** are then written in rows in the **right hand column.**

tens digits go in this column - this is the **stem**

Stem	leaf
3	
4	
5	
6	
7	

units digits go in this column - this is the **leaf**

Look how to write the numbers 56 and 78 in the stem and leaf plot

The first number is 56 the **stem** is 5 and the **leaf** is 6

Stem	leaf
3	
4	
5	6
6	
7	8

The second number is 78: the **stem** is 7 and the **leaf** is 8

The data is arranged on the stem and leaf display as follows:

Stem	leaf
3	2, 9, 6, 3, 9
4	5, 5, 4, 7, 0, 7, 8, 5
5	6, 9, 4, 4, 2, 0, 2, 7, 5, 3, 7, 6, 5, 8, 5, 1
6	5, 9, 6, 1, 3, 2, 5
7	8, 7, 2, 1

When you have entered the stem and leaves into the table, the table are then redrawn with leaves written in ascending order. This makes it easier to read.

Stem	leaf
3	2, 3,6, 9, 9
4	0, 4, 5, 5, 5, 7, 7, 8
5	0, 1, 2, 2, 3, 4, 4, 5, 5, 5, 6, 6, 7, 7, 8, 9
6	1, 2, 3, 5, 5, 6, 9,
7	1, 2, 7, 8

Please take note:

- The **leaf** is the digit in the place furthest to the right in the number.
- The **stem** is the digit or digits that remain when the leaf is dropped.
- So if the list of numbers included 120 ; 134 ; 127 then 12 and 13 would be the stems and 0, 4 and 7 would be the leaves.
- If the list of numbers included a single digit numbers, then the stem would be 0.

Activity 3

Summarise the heights of Grade 8 boys recorded by the Census@School 2009 in an Eastern Cape school measured in centimetres that we looked at earlier in no 2 above, into a stem and leaf display:

a) Rewrite the leaves in ascending order

b) What is the immediate impression you get of the heights of the boys?

Activity 4

The Census@School reveals that the last four digits of the work phone numbers of the parents of the learners in a Grade 9 class in Free State are given below: Write the following into stem and leave.

3315	**3301**	**2205**	**2865**	**2608**	**2886**	**2527**	**3144**	**2154**	**2645**	**3703**	**2610**
2768	**3699**	**2345**	**2160**	**2603**	**2054**	**2302**	**2997**	**3794**	**3053**	**3001**	**2247**
3402	**2744**	**3040**	**2459**	**3699**	**3008**	**3062**	**2887**	**2215**	**2213**	**3310**	**2508**
2530	**2987**	**3699**	**3298**	**2021**	**3323**	**2329**	**2845**	**2247**	**3196**	**3412**	**2021**

Grouped frequency tables

The heights of grade 8 boys in above can also be summarized into grouped frequency table where you can work with ranges of values rather than individual values. In the grouped frequency table the numbers are arranged in groups or class intervals.

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Table 1: grouped frequency table

Height	Tally	Frequency
136	/	1
138	/	1
139	/	1
140	///	3
145	~~////~~ ///	8
146	/	1
147	/	1
150	///	3
152	//	2
154	/	1
155	~~////~~ /	6
156	///	3
157	/	1
158	~~////~~	5
160	~~////~~ ~~////~~ /	11
161	/	1
162	///	3
163	///	3
164	//	2
165	~~////~~ ////	9
167	////	4
170	~~////~~ ///	8

171	/	1
172	/	1
175	//	2
177	/	1
178	//	2
179	//	2
180	/	1
TOTAL		

You can arrange the heights into groups of multiples of ten:

Marks	Tally	Frequency
130 ≤ **cm** < 139	/	3
140 ≤ **cm** < 149	//// ////	10
150 ≤ **cm** < 159	//// //// //// //// /	21
160 ≤ **cm** < 169	//// //// //// //// //// //// ///	33
170 ≤ **cm** < 179	//// ////..////..//	17
180 ≤ **cm** < 189	/	1

Activity 5

Given the Mathematics marks of a class of learners:

32 ; 56 ; 45 ; 78 ; 77 ; 59 ; 65 ; 54,
54 ; 39 ; 45 ; 44 ; 52 ; 47 ; 50 ; 52
51 ; 40 ; 69 ; 72 ; 36 ; 57 ; 55 ; 47
33 ; 39 ; 66 ; 61 ; 48 ; 45 ; 53 ; 57

a) Copy and summarise the mathematics marks into a frequency table:

Mathematics marks	Tally	Frequency
20 – 29		
30 – 39		
40 – 49		
50 – 59		
60 – 69		
70 – 79		

b) Lerato wanted to do a survey about the height of learners in her Grade 12 class. She collected the data below: (the measurements are all in metres)

1.82	1.64	1.71	1.86	1.64	1.67
1.73	1.76	1.84	1.52	1.63	1.65
1.80	1.67	1.71	1.64	1.58	1.81
1.67	1.74	1.69	1.56	1.68	1.74
1.79	1.83	1.69	1.58	1.57	1.73

c) Use groups intervals of $1,50 < x \leq 1,55$; $1,55 < x \leq 1,60$; etc. to draw up a grouped frequency table for Lerato’s data.

d) How many learners are taller than 1.75m?

Activity 6

1. In a class in a school in Mpumalanga, Census@School recorded the time 30 learners spent watching TV in a week. Their answers (to the nearest hour) are given below:

12	**20**	**13**	**15**	**22**	**3**
6	**24**	**20**	**15**	**9**	**12**
5	**6**	**8**	**30**	**7**	**12**
14	**25**	**2**	**6**	**12**	**20**
18	**3**	**18**	**8**	**9**	**20**

a) Draw a stem and leaf display for this data.

b) Complete the grouped frequency table for the data shown below:

Group	**tally**	**frequency**
0 – 10 min	~~||||~~ ~~||||~~ ||	
11 – 20 min		
21 – 30 min		
Total	**30**	**30**

2. The metro bus company in Durban did a survey to find out how many learners used a particular bus to come to school in town. They counted the number of learners on the bus each time it arrived in town. The numbers are given below:

11	**25**	**60**	**58**	**55**	**16**	**23**	**2**	**44**	**26**
49	**8**	**14**	**24**	**7**	**16**	**47**	**5**	**30**	**34**
9	**33**	**10**	**21**	**1**	**56**	**32**	**19**	**6**	**1**
21	**42**	**9**	**35**	**25**	**55**	**37**	**52**	**15**	**7**
31	**25**	**6**							

a) Draw a stem and leaf plot for the data

b) Draw a grouped frequency table for the data. You will need to think of appropriate group intervals.

c) Why do you think the bus company might want to know the numbers of learners on the bus?

3. A survey was conducted to find the colour of eyes of South African learners and the following results were recorded and organised in a table:

Gender	Brown	Green	Blue	Other	Unspecified	Total
Male	60531	1615	2235	5095	704	**70180**
Female	66993	2015	2449	4686	538	**76681**
Unspecified	789	20	33	55	247	**1144**

a) What fraction of all the learners has blue eyes?

b) What is the total number of learners with brown eyes?

Why do you think that there is a high number of learners with brown eyes?

REFERENCES

Statistics South Africa. 2009. CENSUS@SCHOOL. Data handling and probability (Grade 7,8 &9). Pretoria : Statistics South Africa

Upton, G. & Cook. 2001. Introducing Statistics 2nd Edition. Oxford University Press.

FINDING LINKS BETWEEN RATES OF CHANGE OF THE DISTANCE/TIME GRAPH AND AREA UNDER THE SPEED/TIME GRAPH

Maria S. Weitz and Bharati Parshotam

University of the Witwatersrand

TARGET AUDIENCE: Secondary School teachers

DURATION: 1 hours

MAXIMUM NUMBER OF PARTICIPANTS: 30

MOTIVATION FOR THIS WORKSHOP:

Historically, mathematicians and scientists noticed relationships between finding average rates of change and approximating the area under a rate graph. These relationships form the basis for linking different areas of calculus. In this workshop, we will investigate how to represent *change in distance* in different ways on distance-time graphs and on speed-time graphs. A change in distance can be found by *multiplying* (speed × Δtime), by using *area*, or by *subtracting* two distances. Also explore in what ways area calculations use opposite processes to average rate of change calculations. In other words, we will look at:

How can these concepts be shown graphically?

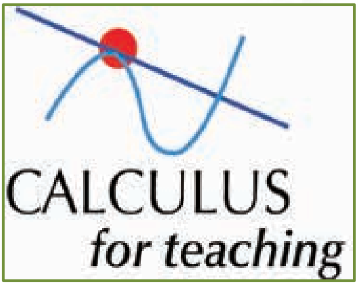
- $\Delta distance \approx instantaneous\ speed \times \Delta t$
- $\Delta distance \approx rectangle\ area$
- $\Delta distance = f(b) - f(a)$

In what way is an area calculation the "opposite" of finding speed?

DESCRIPTION OF CONTENT OF WORKSHOP:

Section	Content	Timeframe
1	Explanation of the link between the rates of change of a distance/time graph and the area under the graph of the speed/time graph	15 mins
2	Task 1	15 mins
3	Task 2	15 mins
4	Discussion and conclusion	15 mins

1 The activities and worksheets to be used in the workshop



ACTIVITY 1: WORK WITH RATE GRAPHS

- ***How can these concepts be shown graphically?***
  - $\Delta distance \approx instantaneous\ speed \times \Delta t$
  - $\Delta distance \approx rectangle\ area$
  - $\Delta distance = f(b) - f(a)$
- ***In what way is an area calculation the "opposite" of finding speed?***

You will now work with distance-time and speed-time graphs to help you answer the questions below.

- How is distance measured on a distance-time graph and on a speed-time graph?
- In what way is an area calculation the "opposite" of calculating speed?

Below are two graphs that show the behaviour of a falling stone. Fig 1 shows the distance ($f(t)$) that the stone falls, and Fig 2 shows the *instantaneous* speed ($v(t)$) of the stone at different times.

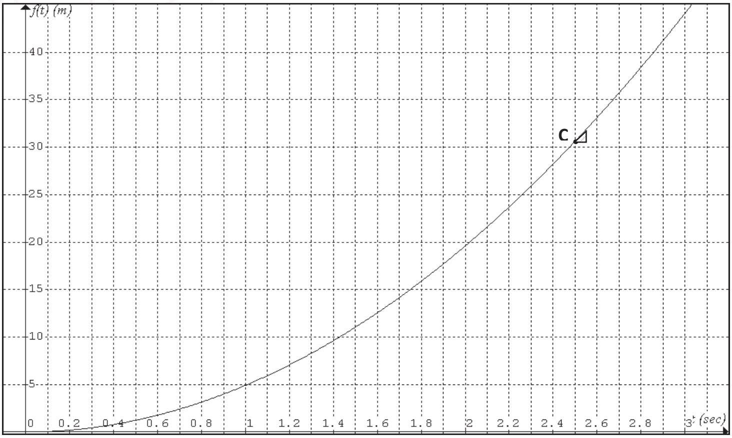


Fig 1: Distance-time graph: ($f(t) = 4{,}9t^2$)

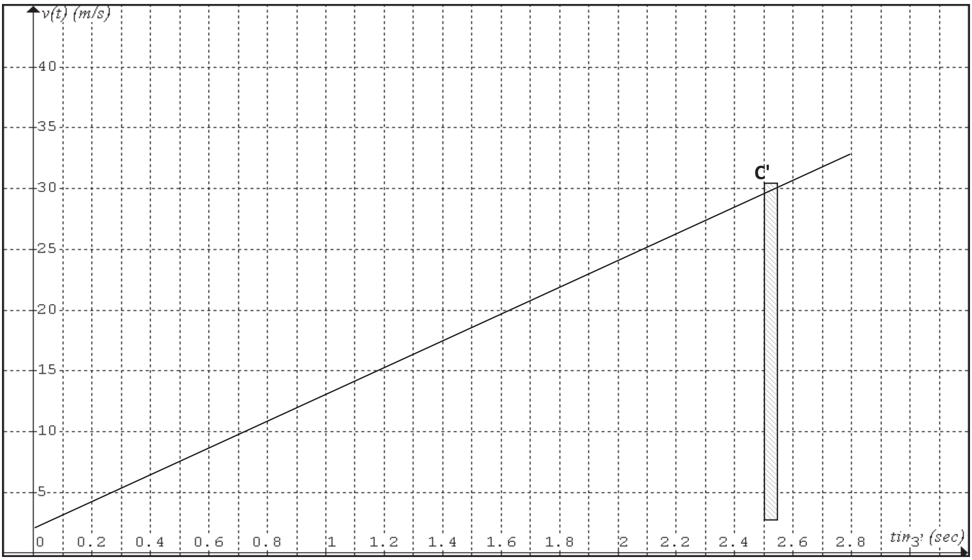


Fig 2: Speech-time graph

This question will help you to understand the graphs in Figures 1 and 2.

a) Both graphs start at the origin. What two facts does this tell us about the stone?

b) Explain what each graph tells us about the stone one second after it is dropped.

c) Explain clearly how each graph shows that the stone is falling faster and faster.

d) Look at points C and C' on the given graphs.

i) Write down the coordinates of both points.

ii) What does the point C tell us about the stone?

iii) What does the point C' tell us about the stone?

ACTIVITY 2: WORK WITH RATE/AREA GRAPHS

In this question, you will see how the two graphs show the same information in different ways.

a) Look at points C and C' on the given graphs.

i) Show this information at point C on the distance-time graph: $\Delta t = 0{,}05$, $\Delta f = 4{,}9(2{,}55)^2 - 4{,}9(2{,}5)^2 = 1{,}23725$, and average speed = $1{,}24 \div 0{,}05 = 24{,}745$.

ii) Show this information at point C' on the speed-time graph: $v(2{,}5) = 24{,}5$, and

rectangle area = 24,5×0,05=1,225.

iii) Give the unit of each value in i) and ii).

iv) Some values in i) and ii) are exactly the same, while other values are only approximately the same. Explain why this is so.

b) Plot the point B(2; v(2)).

i) Shade an area that represents the product $v(2).\Delta t$ if $\Delta t = 0{,}1$, and calculate this area.

ii) What are the units of the product in ii)?

iii) What does your answer in ii) tell you about the stone?

iv) Show this same information on the graph in Figure 1, and use the function $f(t)$ to calculate the value in i).

v) Which gives a more accurate answer: the work in i) or in iv)? Why?

c) i) On the graph in Fig 1, draw in a triangle to show $\Delta y = f(3{,}1) - f(2{,}9)$, and calculate this difference.

ii) What are the units of the length in i)? What does this length represent?

ii) On the graph in Fig 2, shade an area that represents your answer in i). How does the value of this area compare to your answer in i)? Explain.

iii) Use your triangle in i) to approximate the instantaneous speed of the stone after 3 seconds.

iv) Explain how your answer in iv) can be found in Fig 2.

d) i) Use the graph in Fig 1 to find how far the stone has fallen during the third second (from $t = 2$ to $t = 3$).

ii) Shade an area in Fig 2 to show this same distance. Calculate this area.

iii) Compare your two answers.

e) You will now use more than one rectangle area to approximate distance travelled. Use the graph in Fig 2.

i) Draw in left rectangles of width 0,1 under the graph, over the interval [0; 0,5]. Calculate this left sum.

ii) Find the right area sum for rectangles of width 0,1 over the interval [0; 0,5].

iii) Use your calculations to approximate how far the stone falls over the first ½ second.

iv) Explain how you can use the graph in Figure 1 to check your area calculations.

2) In the above work, you sometimes read values off the graphs, and you sometimes did calculations. The calculations usually involved these operations: adding, subtracting, dividing and multiplying.

Explain which operations you used to approximate speed, and which operations you used to approximate the area under a graph.

3) Answer the two questions in the box at the top of page 1.

Note:

For a moving object, consider the distance-time function $f(t)$ and the speed-time function $v(t)$.

Finding change in distance:

- On the *distance-time* graph of $y = f(t)$, the change in distance over the time interval [*a; b*] is given by the vertical distance $f(b) - f(a)$.
- On the *speed-time* graph of $y = v(t)$, the change in distance over the time interval [*a; b*] is given by the area under the graph from $t = a$ to $t = b$.

  * The area of a rectangle under the speed-time graph is given by $v(t) \times \Delta t$.

  * The area under a speed-time graph can be approximated by *adding* rectangle areas.

Finding speed:

- On the *distance-time* graph of $y = f(t)$, the speed at $t = a$ is approximated by the average gradient $\dfrac{f(a+\Delta t) - f(a)}{\Delta t}$.
- On the *speed-time* graph of $y = v(t)$, the speed at $t = a$ is given by the function value $v(a)$.

Comparing the calculations:

- To approximate the area under a speed-time graph, we *multiply and add*: we find the sum

  of $v(t) \times \Delta t$.
- To approximate instantaneous speed, we subtract and divide: $\dfrac{f(a+\Delta t) - f(a)}{\Delta t}$.
- Approximating speed from a distance graph is therefore the opposite of approximating change in distance from a speed graph.

HOW I TEACH TALLY TABLES IN THE CONTEXT OF THE CYCLE OF DATA HANDLING

Leonora Joy Davids
Tuscany Glen Primary School, Bluedowns, Eerste River, Cape Town

INTRODUCTION:

This presentation will be about how I teach tally tables in Grade 4. I have chosen this topic because Grade 4 is the first time that the learners are required to draw up tally tables. Another reason for choosing this topic is because teachers and texts often teach it in isolation, and not in the sequence that it should be taught – whch in in the context of the cycle of data handling.

CONTENT

A brief video clip will be shown.

*** play clip (humerous YouTube clip about teaching - 1 minute)

When I started teaching, I just did whatever the textbook said to do, only to find out, four years down the line, that textbooks don't always give you the best way of teaching something.
I looked at the ANA test my grade 4's wrote in September, and saw that even there, they "put the cart in front of the horse".

*** Show and discuss example from ANA 2013 Grade 4 test.

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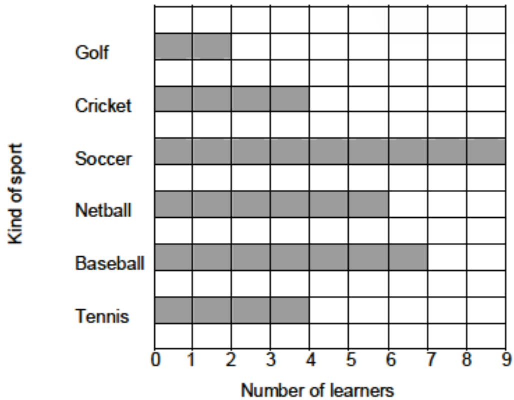
18. This bar graph shows the most popular kind of sport amongst the learners in Grade 4.

LEARNERS' FAVOURITE SPORT



18.1 Complete the tally table.

KIND OF SPORT	TALLY MARKS
Golf	
Baseball	
Tennis	

(3)

This brings me to the question: "What is the purpose of doing tally tables?" We can clearly see here that this test creates a misconception as to the purpose of tally tables. Why do we make tally tables? Do we want to see that the child knows how to draw up a tally table? Do we want to see if children can make groups of fives? Or is it simply to add marks to our question/test papers?

That this test item requires learners to make tallies from en existing bar graph, creates an impression that tally tables are arbitrary. Even if this is a testing context. Tests, even when not intended, can be formative, and can direct learners' understanding of some aspects of school mathematics content.

I think a tally should actually be the basis of the other graphs – you should get your information from the tally to complete your bar graph, pictograph, etc.

Why are tally tables important? The reason we do tally tables is to sort out raw data; to sort out a mess in a systematic and organized way, without getting confused easily. This is especially if we are working with a huge data set.

If I had to find out from about 50 respondents, what their favourite soft drink was, choosing from Coke; Fanta; Sprite; Stoney; Appletizer, I would need to do the following:

- Find a way of recording my data (without worrying about organizing it – yet)
- Once I've recorded my data accurately, how can I organize it?
- Once I've organized it, how can I represent it in a way that the data can be easily read?
- What does my data tell me, and which further questions arise?

*** Work through the above steps, getting the required information from the participants.

- The preferences, in no order, are written on the board/flip sheet
- The data is systematically organized from the recorded raw data onto a tally table
- The organized data is now represented on a bar graph

I would ask as to how useful this data is. How can it be used?
The same way in my class, I must have reasoning behind why we are gathering information. I always try to collect data in my class that can lead to some sort of decision, or inform an upcoming event, like the class party, an outing, or any other decision that could influence the all the children in the class.

Another "danger" for when deciding on which information you wish to collect, is that you (or the learners) can come up with too many options. For example, if you ask: "What's the most popular colour in this class?" you may come up with too many variations and preferences (red, pink, light pink, maroon, mauve, indigo, purple, etc.) It's a good idea to give them a limite choice from a list. (Choose your best colour from the following list: red, blue, green, orange, purple.)

So what "themes" can we use to collect data in the intermediate phase classroom?
Some of my personal examples are:
Fun day events (What's the most popular stalls to have?)
Fast foods (What's the most delicious fast food? Choose from…)
Class party (Which cool drinks must we buy? Choose from…)

*** Practical exercise (depending on number of attendees)….
There are three types of lunches here today, which one would you prefer?

- Chicken burger
- Beef burger
- Rib burger

The participants can now organize themselves and collect the raw data, make tally tables and then draw bar graphs from the data that they've collected.

CONCLUSION

We will conclude with a discussion on tally tables and the role that they played in organizing the data during the activities. Participants will be encouraged to reflect on the activity. We will exchange ideas on topics and resources that could be used and how to make the learning experience fun for the learners.

REFERENCES

ANA Test 2013 (Mathematics Grade 4)
Course notes (ACE Curriculum Leadership Languages and Mathematics 2013)
Own Experience

KEEP CALM – DO MATHS ON AN IPAD

Laetitia de Jager

Think Ahead Education Solution

Learners' phobia for Mathematics has led to many learners dragging their feet to the Mathematics class and barely passing their Mathematics tests. Research has shown that learners' fear for Mathematics begin when they're still in primary school. Join me in the iSchoolAfrica journey of using iPads to change the way we teach Mathematics and demystify the learners' perception of Mathematics in the classroom.

DEMYSTIFYING MATHS – ON AN IPAD

Why is it that the majority of our high school learners don't like Mathematics and opt for Maths literacy in Grade 10 rather than Mathematics as their subject choice? Why is it that most adults will tell you how they hated Maths at school and that they're really not any good at it?

Human beings all have irrational fears and anxieties about everyday objects and situations like spiders, airplanes, enclosed spaces, heights and.... maths. Grown-ups admit to hating maths, or being really bad at it. Scientists say that this dislike of Maths may be disguising a real phobia that probably begins at an early age.

Stanford researchers found proof that Maths anxiety is a real thing, certain people seem prone to feeling real anxiety when it comes to doing Math, and that anxiety in turn hampers their ability to do math in the first place. They got their proof from studying grade 2 and 3 learners and performed MRI's on them while they did simple Maths problems. They found that the grade 2 and 3 learners were faced with simple addition problems, the parts of their brain that feels stress lit up, and the parts that are good at doing math, deactivated. Interestingly they noticed that the children with math anxiety weren't actually bad at Maths - they got the same number of answers right as their anxiety-free peers, but it took them longer to solve the problems.

The good news according to researchers is that phobias are treatable, which means there may be a way to cure kids' anxiety before they develop a lifelong aversion to Maths.

Vinod Menon, a professor of neurology and psychiatry who led the Stanford study explains:

> *"We already have mechanisms that are widely used to treat phobias. If the same brain system is involved with maths anxiety, you should be able to use those mechanisms.*

What the math field might want to think about is doing this at earlier ages. Once Maths become a strongly learned fear, then it's much more difficult to treat."

The big question is just: How do we do this?

Hung-Hsi Wu from UC Berkely, professor in Mathematics who has written a book about teaching math to elementary school students explains 'demystifying maths':

"*You're forced to do something over and over again and you know nothing about it - wouldn't you be anxious too? I believe almost all math anxiety would be gone if every one of our teachers in elementary school was capable of looking in the 'black box' of mathematics and telling kids there is no mystery, this is why it works"*

What is this 'black box' he is referring too? Well, keep calm and see how we help teachers to do demystify maths for their learners using an iPad.

In this session we will show you how we use iPads in the classroom to transform the way mathematics is taught in schools and how by implementing the SAMR model (by Ruben R Puentedura) teaching of Maths with iPads transforms learning Maths in ways never conceived before. The following table takes us through the journey of demystifying maths for learners using iPads in their classrooms.

Example of learning progress when doing simple maths functions on an iPad	**Substitution** *iPad acts as a direct substitute for traditional teaching tools*	**Augmentation** *iPad acts as a direct tool substitute, while using a technology function/app*	**Modification** *iPad allows for creativity and invention using a combination of apps for new outcomes*	**Redefinition** *iPad allows for collaborating and learning in ways never imagined before*
I interaction	Learners use iPads to answer maths problems they could've answered on a worksheet or in a workbook e.g. mental maths. No or very little interaction between peers or teacher and learners	Learners are using apps to practice mental math using an app e.g PopMaths.	Learners are able to use productivity apps and/or a combination of apps to explain a maths concept.	Learners use the iPad for all basic learning principals, but move one step further by inviting other participants in his/her learning process e.g. research studies, content apps, productivity apps, podcasts etc.

P **Participation**	No or very little interaction between peers or teacher and learners.	Learners have immediate feedback if an answer is wrong and can correct. Collaboration between peers is evident when learners help each other to find answers to problems. Apps are also fun to use and the learners' engagement becomes significant in the process.	Learners collaborate and use each other's efforts to build a project where new and different ways of finding the answers to problems are sought. Learners rely on each other to find the why's in their journey to solving problems.	Collaboration happens between the learner and his/her peers, teachers and any other resources available. The learner now takes the lead in his learning process, but makes use of all other resources to find the why's in his journey to solving problems.
A **Advancement**	Learners might improve their mental math much, but they are learning to answer problems in a different context	Because of the immediate feedback, learners are able to assess their own progression. They have the ability to improve their marks though being positively engaged with the content on the iPad.	Learners are not given the formulae and steps, but are expected to understand the process followed by intuitively working together. The iPad allows learners to be creative and actively take part in their own learning process.	Learners are purposefully finding their own steps to solve problems by looking at other sources and using their own intuition and creativity. They are able to use their strong base of mathematical knowledge to be inventive
D **Demystifying**	Very little has changed in the learner's mind about the challenge of Maths.	Learners are having fun in a non-threatening environment, which is the first step in overcoming fear and anxiety.	Because learners are engaged and a co-designer of their learning process, they are able to internalise the learning and find the whys behind the hows when purposefully solving problems.	Learners are not only driven to find answers to problems, but because the learning process becomes a journey of discovery that explains real life situations, explained by the language of maths without the mystery

Table 1: Using the SAMR model to explain the process of using iPads in the Maths class

REFERENCE:

Blog: http://hippasus.com/rrpweblog/

http://www.sfgate.com/education/article/Demystifying-math-could-ease-anxiety-3466413.

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HOW I TEACH FRACTIONS IN THE INTERMEDIATE PHASE WHEN THE LANGUAGE OF LEARNING AND TEACHING (LOLT) IS ENGLISH AND THE LEARNERS' HOME LANGUAGE IS NOT ENGLISH

Nombeko Mafenuka, Janine Wilson

Pearson South Africa

In 2004, Myburgh et al. reported that where learners do not speak the language of instruction, authentic teaching and learning cannot take place. The Department of Basic Education agrees and states that such a situation largely accounts for the school ineffectiveness and low academic achievement experienced by students in Africa (DBE, 2010). We feel that since the LOLT in Intermediate Phase Mathematics is English in many schools in South Africa, it is important that teachers have the necessary tools to support and assist their learners in this crucial phase.

INTRODUCTION

In this lesson we focus on fractions. We believe that introducing English terminology on fractions in Grade 4 should be done slowly, with plenty of repetition and with the use as many interesting visual resources as possible.

SIX THINGS TO CONSIDER BEFORE YOU START A LESSON

1. Make sure you explain to learners why it is important to learn about fractions. Use pictures or props wherever possible. For example:

- Fractions are important because we encounter them in our everyday lives:
    - Fractions are used in time, such as quarter past, half past, half a day, quarter of a year. Show them a clock face to illustrate some of these times.
    - When cooking and baking we measure ingredients using words like: half a dozen eggs, half of a tablespoon, a quarter of a cup.
    - In shops we often see signs that say ‘Half price sale!’

2. Reinforce vocabulary in order to boost their language development. For example, go through these words one at a time explaining each one and illustrating each word using pictures or drawings wherever possible:

whole number, half, double, quarter, third, tenth, divide, share, equal parts, denominator, numerator, common fraction, proper fraction, mixed fraction, equivalent fraction

3. Use visual resources that will make it easier for the learners to understand the concepts you are trying to explain. For example:
    - laminated fraction strips
    - different coloured shapes cut into fractions and labelled
    - fraction walls and number lines
    - flash cards with fraction notation and words
    - pictures of everyday objects that can be divided into fractions.
4. Explicitly teach thinking skills rather than have learners rote learn the subject. Remember that learners should be exposed to different types of questions that will make them think creatively, even if they haven't yet mastered the language. For example:
    - '54 ÷ 6 = 9' **or** '54 sweets shared between 6 children'.
    - '54 ÷ 2 = 27' **or** 'Halve 54' **or** 'What is half of 54?'
    - Ask learners: 'Why are $\frac{1}{2}$ and $\frac{5}{10}$ the same?' Instead of asking them 'Which fraction is equivalent to $\frac{5}{10}$?'
5. Construct a multiple meaning bank that will help learners know a question whichever way it is been posed. For example:
    - 'What is one fourth of twelve?' **or** 'What is a quarter of twelve?'
6. Use collaborative learning strategies to help learners to learn from each other. For example:
    - The way your learners sit in class should be determined by the kind of activity you will be doing. If you have been teaching a lesson that is difficult for some learners to grasp, mix those who struggle with those who have understood the lesson well and let them help each other.

THE LESSON

Lesson objectives

***NB:** You may not get through all of these objectives in one lesson. Be aware of whether learners are keeping up and teach at the correct pace for your learners.*

At the end of the lesson learners will be able to:

- understand prerequisite knowledge using new terminology in English, i.e. sharing and fractions

- define fraction, numerator and denominator (in English)
- identify the number of shaded parts and the number of equal parts in a shape
- recognise and write a fraction using mathematical notation and using words
- determine if two fractions are equivalent using shapes
- add fractions with the same denominators.

Starting off

Show learners a bar of chocolate (or any other physical resource that can be divided in different ways) and ask them: 'If I have one bar of chocolate and I want to give it to two children, what should I do to make sure that each child gets an equal share?' Illustrate these using two learners in the class, or using pictures or drawings.

Then discuss: 'If I want to give the same bar of chocolate to three children, how will I make sure that each child gets an equal share?'

And lastly: 'If I want to give the same bar of chocolate to four children, how will I make sure that each one gets an equal share?'

Write down answers as they give them to you.

Once you've written all the answers down, ask the learners how they worked out their answers and discuss them together. If learners do not easily come up with the correct answers, make representations of the chocolate using cardboard or paper, cut the drawing up into pieces and share the pieces between the children.

Class Activities

1. Divide your learners into groups and give each group one bar of chocolate (or any other physical resource that can be divided in different ways). Make sure the chocolate can be divided equally amongst the number of children in the group but try to have groups of different sizes (for example, if you have bars of chocolate that have 12 pieces, divide the learners into groups of 2, 3, 4 and 6). See how they share the chocolate you have given them. Make sure that each child in a group gets an equal share of the group's chocolate.

Then ask them to look at the pieces they have shared and talk in their groups about what fraction each member of the group has. Then ask different groups to compare how the chocolates have been divided differently. Discuss who got the biggest portion and why. (This will be determined by the number of learners in the group – if there were many learners in a group, each learner will get a smaller fraction of the chocolate.)

Ask one learner from each group to come to the front of the class with their pieces and then ask the rest of the learners to arrange them from most to least (biggest fraction of the whole to smallest fraction of the whole), according to the pieces they have in their hands. Let them discuss which fraction is bigger and which fraction is smaller.

2. Write a fraction on the board, or hold up a fraction card for everyone to see and ask them if they know what this is called. (For example: $\frac{1}{2}$)

Expected answer: Fraction

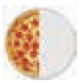
Then ask them to explain what a fraction is.

Expected answer: A fraction is a part of a whole.

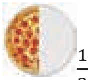
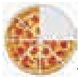
Tell learners that in fraction we have two numbers. Ask them if anyone can tell you what these numbers are called.

Expected answer: The top number is called the numerator and the bottom number is called the denominator.

3. Show learners some shapes with shaded parts and ask them to tell you which fractions they represent. Make sure that learners can match the fractions in fraction notation and also in words. For example:

					
$\frac{1}{2}$ one half	$\frac{1}{4}$ one quarter	$\frac{3}{8}$ three eighths	$\frac{2}{4}$ two quarters	$\frac{4}{8}$ four eighths	$\frac{3}{4}$ three quarters

Ask your learners to look at the fractions below and say which ones they think are different but have the same value. Tell them that these fractions are called equivalent fractions. Give them a chance to choose those fractions that are equivalent to each other. For example:

 $\frac{1}{2}$	 $\frac{1}{4}$	 $\frac{3}{4}$
 $\frac{2}{4}$	 $\frac{2}{8}$	 $\frac{6}{8}$

4. Ask your learners if they know how to add fractions together. Go over the difference between the numerator and the denominator of a fraction again. Then explain that they can only add fractions together when the denominators are the same. Show them physical examples or drawings so that they understand why this is the case. Then show them this example:

$$\frac{5}{10} + \frac{3}{10} = ?$$

The denominators are the same, so we can add the numerators.

- Add the top numbers and put the answer over the denominator.

  $$= \frac{5+3}{10} = \frac{8}{10}$$

  Illustrate this by counting fraction pieces, colouring in drawings as you count, or ‘hopping’ in eighths on a number line. Do not expect learners to grasp the notation straight away. Use many examples to show them how to do this.

ADDITIONAL NOTES AND TIPS

- Explain terms in English using simple language wherever possible.
- Use as many visual resources as you can, including physical resources that learners can manipulate using their hands.
- Use a class test to find out what learners know and what they don’t know, and immediately intervene as soon as you discover problems.
- Talk to the parents of learners who are struggling. The best is to call a group of parents in and show them how to help their children at home. If they are unable to help, advise them to ask trustworthy friends, family members or even an older learner to help their child after school.

- Encourage learners to talk about how they got to their answers so that you can easily find mistakes and misunderstandings they may have.
- Be sensitive and take extra care considering those learners who find fractions difficult and frustrating. Always be patient with these learners. It is much better and easier for you if learners love the subject rather than hate it.
- Praise learners for each little improvement they show and always tell them how great they can be if they practise working with fractions everyday.
- Many learners and teachers think of fractions as the most difficult area to understand in Mathematics. Try to keep your lesson positive and make sure that learners believe that they **can** do it. Always try to make learning fractions fun and easy.
- Try to bring sweets, chocolates, pizzas or biscuits to work with in class, and let learners share them at the end of the lesson.

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Department of Basic Education. 2010. The status of the language of learning and teaching (LOLT) in South African public schools.

Myburgh, O., Poggenpoel, M. and Van Rensburg, W. 2004. Learners' experience of teaching and learning in a second or third language. Education, 124(3): 573.

FORMAT ON AN ANALOGUE CLOCK- GRADE 5

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St. Augustine's Primary School

INTRODUCTION

In many Western Cape schools, learners are under-performing in the WCED Grade 6 Systemic Tests. The WCED LITNUM results of 2012 (Table 1) shows that only 26.4% of the Grade 6 learners were able to score 50% or more. In contrast, the Grade 3's achieved a pass rate of 51.5%. In this regard, learners generally perform most poorly in MEASUREMENT. Why the drop in learner performance between Grade 3 and Grade 6? We are of the opinion that the drop in performance may in part be attributed to the fact that teachers in the Intermediate Phase (IP) are too quickly adopting teaching strategies that too heavily rely on abstract thought. Our intent of this presentation is to show that, even at the IP level, the pedagogic situation should incorporate the use of concrete apparatus and to give an indication of learner performance in this regard.

Mathematics			
	% pass rate in 2011	% pass rate in 2012	Diff. 2012/2011
Grade 3	47.6%	51.5%	+ 3.9%
Grade 6	23.4%	26.4%	+ 3%
http://wced.school.za/comms/press/2013/7_14jan.html			

Table 1: 2011/2012 WCED LITNUM results

CONTENT

We will then demonstrate how we used various resources to facilitate reading time in 12-hour format on an analogue clock. The activities include:

- A mental exercise on the previous knowledge of the learners regarding time, using questions such as: How many days are there in one year? How many months in a year? How many weeks in a year? How many hours in a day, minutes in an hour, seconds in a minute, etc.?
- Some conversation about the sundial, minute glass and other objects used to tell time e.g. the candle (when daddy came to visit mommy before they were married) was called the "opsitkers";
- The use of diagrams to discuss the ideas 'to the hour' and 'past the hour'

- Introducing an analogue clock (see Figure 1) from the Maths, Science and Technology Kit supplied by WCED - to indicate why it is called analogue & the function of the 'hands'
- How various times are read on the analogue clock, demonstrating how the learners are explicitly and continually involved in lesson.

The worksheet, on which they have to record times indicated on the 'clocks' on the worksheet, will then be discussed

Consolidation activity:

To consolidate the lesson, we play a Loop game that consists of 26 cards (see examples below – Figure 2). These are available on the www.superteacherworksheets.com website. Each card has a representation of a clock indicating a different time, with the time written below. One learner will say "I have ….." and read the time on the clock and then ask "Who has ….?" and then read the time at the bottom. This will continue until all the cards have been read and therefore each learner will have a turn to read his or her time. The answer chain (Figure 3) is shown below.

CONCLUSION

Some outcomes of the lesson:

- learners overcame a general problem where they say "45 minutes past 2 o'clock", instead of "15 minutes to 3 o'clock"
- learners displayed greater confidence in reading analogue time
- learners were able to draw the conclusion that 'adding 12 to each number on the clock' gave the 'pm' times - a consequence of the numbers 13 to 24 on the clock face

The Loop game can be played at any time throughout the year - as a practice activity.

Figure 1

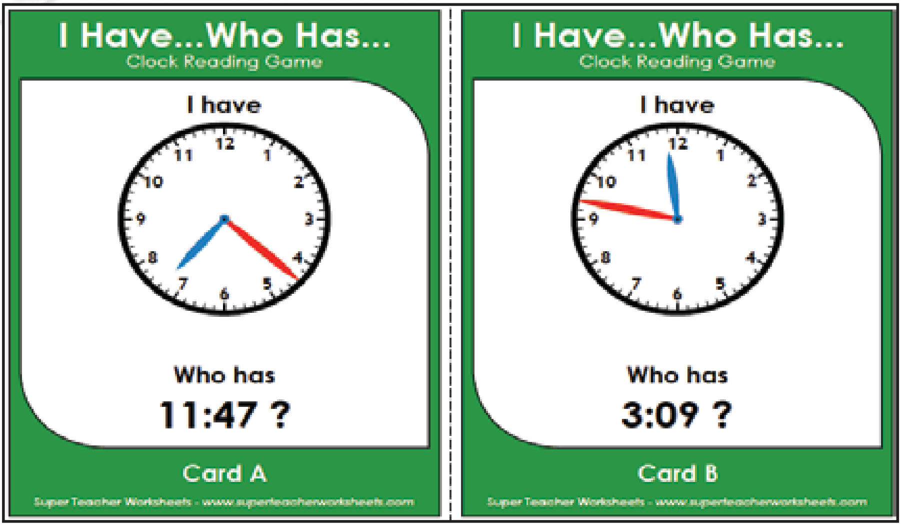

Figure 2

Answer Chain:

A: I have 7:22. Who has 11:47?

B: I have 1:47. Who has 3:09?

C: I have 3:09. Who has 8:35?

D: I have 8:35. Who has 1:17?

E: I have 1:17. Who has 6:42?

F: I have 6:42. Who has 11:29?

G: I have 11:29. Who has 4:45?

H: I have 4:45. Who has 9:39?

I: I have 9:39. Who has 2:06?

J: I have 2:06. Who has 7:56?

K: I have 7:56. Who has 12:30?

L: I have 12:30. Who has 5:24?

M: I have 5:24. Who has 10:43?

N: I have 10:43. Who has 3:33?

O: I have 3:33. Who has 8:14?

P: I have 8:14. Who has 1:48?

Q: I have 1:48. Who has 6:10?

R: I have 6:10. Who has 11:13?

S: I have 11:13. Who has 4:59?

T: I have 4:59. Who has 9:07?

U: I have 9:07. Who has 2:00?

V: I have 2:00. Who has 7:39?

W: I have 7:39. Who has 12:54?

X: I have 12:54. Who has 5:50?

Y: I have 5:50. Who has 10:21?

Z: I have 10:21. Who has 7:22?

Figure 3

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HOW I TEACH MEASUREMENT: LENGTH

Ms Andrea Henderson
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I would like to talk about how I teach measurement (length) by consolidating conceptual knowledge around concepts like length and breadth/width and also giving the children practical activities to do to cement their understanding. The children often do not know what 'estimate' means. They tend to struggle when it comes to using non-standard and standard units to measure. For example, when asked to use a pencil to measure their height, they have no idea how to use the pencil and keep a finger on the end and then move the pencil down. It seems they have not had enough opportunities to measure using non-standard and standard measures, alternatively they had not picked up what to do in the classroom when their class teacher did measurement. This happens when the children are weak and take time to pick up new concepts.

INTRODUCTION

I would like to talk about how I experienced teaching length to children attending learning support in the Foundation Phase. I am a learning support mathematics teacher who teaches the weakest of the weakest in our school. I take groups of 4, or 5 or 6 children at a time for 50 minutes and they attend my classes twice a week. Our children come into the school at Grade R level and most are isiXhosa or Afrikaans speaking. However, the LOLT (Language of Learning and Teaching) of the school is English and the learners are immediately immersed in an English environment. Naturally there are problems when we come to teach mathematics as the children have to learn the vocabulary and concepts of mathematics in a second language. I teach Grade 1 to Grade 4 remedial mathematics.

UNDERSTANDING DISTANCE CONCEPTS

Last year I taught length for about 3 weeks. The first thing I noticed was that the children did not know simple concepts like length, breadth/width, taller, shorter and estimate. I used the door mat in the classroom and we pretended that it was a swimming pool. Most of the children had been to a pool. Then I asked the question: "When you swim from here to here, what do we call this distance?" They replied "We swim the length of the pool".

We then discussed length and breadth/width. The children went into the passage outside my classroom. “Let’s walk the length of the passage,” I said. In this way they came to understand basic language necessary for measuring length.

We are required to first estimate and then measure but the children did not know what ‘estimate’ meant. One child said it was ‘guess’ and so I taught them to first guess/estimate how many measures they would use and then they would measure with a stick, or a body part or ruler or tape measure. The secret was to keep the activities practical and use the environment around them to talk about conceptual language. In other words, I used hands on activities and concrete materials. We went out into the passage and used paces to measure the length and breadth of the passage.

It was identified that children had difficulties understanding the concept of starting from a fixed point when dealing with length (O’Keefe and Bobis, 2008). Another difficulty was that children struggled with the concept of leaving no gaps between units of measurement. When using rulers they sometimes were confused with inches and centimetres on the different sides of the ruler. And where to start? Many began at the end of the ruler and not at the 0. For example when measuring their height with a pencil, they did not realise that they had to start with the pencil point and finish with the eraser and then put the point under the eraser. They were not aware of the importance of accuracy and did not realise that a single unit is an exact repeat of the next unit. I am sure they were taught by their teachers, but they obviously had not had enough practice. I had to show them individually how to measure using a pencil or using a ruler or tape measure. However, they did improve over the period of three weeks and eventually could measure more accurately.

LESSON INTRODUCTION

How did I introduce the lesson? I told a story about a cave family (and made it exciting while telling them that Daddy went hunting and Mommy went to gather nuts and roots). Big Daddy decided one day to move out of their cave and build a house. But what could he used to measure the sides and foundation of his house? The children mentioned body parts and we began to measure using body parts. They mentioned sticks and stones and we measured using sticks. In Grade 2 after using non-standard measures, I introduced the metre.

We did not have a metre stick but I measured metres using wool and allowed them to measure metres using the wool. In Grade 3 I revised metres and also introduced centimetres.

By walking the length of the passage outside the classroom, they eventually came to a rudimentary understanding of metres. I gave them worksheets to complete where they had to estimate and then measure using the wool (1 metre per piece of wool) and the ruler. When I tested the Grade 4 class recently (this year), they were unable to tell me which was bigger – a centimetre or a metre, a metre or a kilometre. Last year we spoke about the distance from the school to the Hypermarket and it was approximately one kilometre. This means they still have not grasped the concept of how long a metre or km or centimetre is. They really do need a lot of practice in order to understand the basic units of measurement.

Because I have small classes, I was able to assist these children to measure accurately. After three weeks (two periods per week) they still had not fully grasped all the vocabulary and concepts needed to prepare them for Grade 4. Our children at our school will need further help in this area. According to the systemic test results, measurement is the one outcome which is most neglected and needs mastery. I believe we need to therefore spend a longer time consolidating measurement in the Foundation Phase.

ENRICHMENT

For fun, and to use more practical activities, what about giving the children a bean to plant and after a couple of weeks allow them to measure the length of the stem? For extension get them to estimate how far it is around the school and take them for a walk and use a trundle wheel to measure how many metres? As I mentioned, our school is approximately 1 km from the Hypermarket. If you have the opportunity to walk to a place like this take the children and tell them they have walked 1 km. Also ask the children what unit of measurement they would use to measure a pencil, the width of the classroom, the length of the passage. This was also identified as a stumbling block in my class. After some practice, the children still did not know which standard unit was to be used to measure different lengths.

CONCLUSION

In conclusion, it seems that children in the Foundation Phase need to spend more time on mastering length. This is especially at our school where the children do not learn in their mother tongue. Concepts like ‘length’, ‘breadth’ and ‘estimate’ need to be mastered before the children can begin to measure. And learners need to learn to measure accurately.

REFERENCES:

O'Keefe, M and Bobis, J. (2008). *Primary teacher's perceptions of their knowledge and understanding of measurement. Proceedings of the 31st Annual Conference of the Mathematics Research Group of Australasia.*

HOW I TEACH WORD PROBLEMS IN A GRADE 9 CLASS

Wandile Hlaleleni
Butterworth High School

Learners perform poorly in word problems. I have spoken to more than fifty teachers of various education bands they all told me that their learners do not do well in word problems .The teachers attribute the poor performance to language , they claim that the learners are unable to translate the verbal representations of word problems to symbolic representation.

INTRODUCTION

I want to share an important teaching approach with teachers, that of using **strategic competenc**e. Strategic competence is one of the strands of mathematical proficiency and it simply means ability to formulate, present and solve mathematical problems. I use this approach when teaching word problems.

HOW DO I TEACH WORD PROBLEMS IN GRADE 9

I usually formulate, present and solve problems with learners. We (the learners and I) usually use the data from our context to formulate problems.

For instance, Sisanda is 15years old and Lindiwe is 18 years old.
We compare the two ages and add them and use that to formulate the following problems.

> *Lindiwe is three years older than Sisanda. If the sum of their ages is thirty three years. How old is Sisanda*?

Solution

(1) Take away more and bring back method (arithmetic method)

33 years – 3 years = 30 years
Since they are two divide 30 years by 2 years to get 15 years (30 years ÷ 2 = 15 years)

Sisanda is 15 years old. To get Lindiwe’s age we add 3 years to Sisanda’s age

(2) Algebraic method (symbolic)

Let Sisanda's age be represented by k and that means Lindiwe is (k + 3) years old. This implies that k + k+ 3 = 33
$2k + 3 = 33$
$2k = 33 - 3$
$2k = 30$
So $k = 15$
Sisanda is 15 years old.

We also formulated a second problem using the same data (their ages) . Since 18 years divided by 15 years give us 1,2 or $1\frac{1}{5}$ and we also observed that that in three years to come Lindiwe will be 21years old and Sisanda will be 18years old and 21years divided by 18years is equal to $1\frac{1}{6}$. We used the relationships as established above to formulate the following problem.

> *Lindiwe is $1\frac{1}{5}$ times as old as Sisanda . In three years to come Lindiwe will be $1\frac{1}{6}$ times as old as Sisanda. How old is Sisanda?*

Solution

Let Sisanda's age be (d) years and that means Lindiwe's age is $\frac{6}{5}$d. (why?)
In 3 years to come Sisanda will be (d + 3) and Lindiwe will be $\frac{6}{5}$d +3
NB: $1\frac{1}{6}$ will be used to make the unequals equal ie to form an equation (why?)
$\frac{7}{6}(d + 3) = \frac{6}{5}d + 3$
$\frac{7}{6}d + \frac{7}{2} = \frac{6}{5}d + 3$

L.C.M. =30
$30(\frac{7}{6}d) + 30(\frac{7}{2}) = 30(\frac{6}{5}d) + 30(3)$
$35d + 105 = 36d + 90$
$-d + 105 = 90$
$-d = -15$
$d = 15$ Sisanda is 15 years old

CONCLUSION

One can use any two ages to engage with problem formulation , presentation and solving. Other word problems could be formulated using money, numbers, measurements or any other interesting contexts.

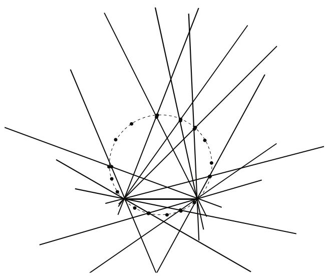
AN EXPLORATION OF CIRCLE GEOMETRY BASED ON ANIMATED NICOLET FILMS AT WYNBERG SECONDARY SCHOOL

Yusuf Johnson
Schools Development Unit (SDU), University of Cape Town
Gavin Nieuwoudt
Wynberg Secondary School

CAPS has brought Circle Geometry back into the core Mathematics Curricum (in particular, Grade 11). The various theorems and proofs connected with this content bring opportunity for investigation, conjecture, formulations of conjectures, (logical, systematic) argumentation and informal-to-fomal proof. The approach is based on short animated films developed and produced by Jean Nicolet, a Swiss school teacher who explored geometrical ideas through black and white silent animated films from 1944 until his death in 1966. This presentation reports on an exploration of the approach with 50 learners at one of the SDU's GrassLow Park Project schools, Wynberg Senior Secondary School (on 26 February 2014). The GrassLow Park Project focuses on teacher development in mathematics content, with classroom support, across senior phase grades, but with extension above and below this phase.

DISCUSSION ON NICOLET AND CIRCLE GEOMETRY

An initial (10-minute) activity will be introduced to the participants. This will form the basis for discussion of the approach. Apparatus will include a pair of intersecting lines on transparency and tracing paper; these will be used in combination with short movie clips (animations) and apps for further demo purposes. In this very brief presentation, most of the high school circle geometry will be surveyed. Practical classroom suggestions and sample learner work, comments, etc. will also be shared with the audience. A full workshop is possible if there is interest.



HOW I TEACH CONGRUENCY

Adrian Lotter
Kannemeyer Primary School, Western Cape

This 'How I Teach' presentation demonstrates how the topic Congruency *is taught with the integration of* Transformation Geometry *and* Measurement*. The pace dictated by CAPS has challenged me to find ways to integrate different topics to develop a sense of continuity for learners.*

INTRODUCTION

Engaging CAPS for the first time in 2014 has been a challenging experience. I find that the sequencing of the content and the concepts is fragmented and could result in there being no integration between content strands. The pace is fast and learners do not always manage to grasp the concepts in the given time. Being faced with the idea of not finishing the term's content, I had to look for innovative strategies to help me complete everything without disadvantaging the learners. Without undermining CAPS in any way, I became more conscious of the need to link mathematical concepts. I experienced success with linking Congruency and Similarity to Transformation Geometry and Measurement. The response of learners to this method was most encouraging.

RATIONALE

In Grade 7, under the topic Transformation Geometry, learners had to recognise, describe and perform translations, reflections and rotations and also draw enlargements and reductions of geometric figures (Senior Phase Mathematics Curriculum and Assessment Policy Statement, 2011, page 29). Translations, reflections and rotations preserve size and shape of polygons while orientation changes. This is a powerful way to illustrate that congruency pertains to the preservation of every part of a polygon even though its orientation might change. Enlargements and reductions relate to the concept of Similarity. By using this integrated method, various other mathematical knowledge and skills are also incorporated:

- Properties of 2-D shapes
- measurement (protractor, ruler)

Ten hours is allocated to the topic, which translates into 10 lessons. However, part of the topic is devoted to the solution of problems. I set aside 5 lessons for teaching Congruency and Similarity.

CONTENT

Learners are challenged to move from exploration and observation to accurate measurement and conclusion. They also shift from using scale drawings to interpreting drawings that are not drawn to scale. This is important, because they will be confronted with drawings that are not drawn to scale when they engage with problem solving.

Day 1

I experienced some difficulties with my first activity, which involved dotty paper and drawing congruent shapes that were transformed. The fact that the learners had difficulties with the construction there of, impacted on achieving the outcomes around congruency. I had also made some assumptions regarding learners' ability to use dotty paper appropriately. Many were not focused on using the measurements to ensure that the sides of their second shape measured exactly the same as the first. The lesson was diverted to cover the purpose of using dotty paper or other grids. This focused learners on the importance of accuracy when comparing geometric shapes.

Day 2

Learners were provided with sets of geometric shapes on coloured paper. Their task was to cut out the sets of shapes so that they had three of each in different colours. They had to create columns where they pasted each shape as a reflection, translation and rotation respectively. There was a focus on measuring the *distance* from the axis of each vertex, particularly for the reflection. Learners had to label each vertex of the translated shape appropriately.

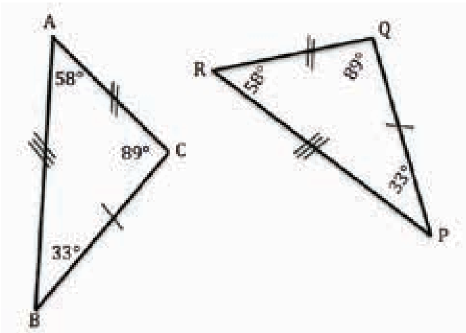
Day 3

This lesson started with a consolidation of the concept of congruency. Learners were handed a worksheet with sets of congruent polygons in different orientations. They needed to use their maths sets to measure sides and angles, and to add the conventional indicators for equal lengths and angles. Based on their measurements, they needed to conclude which shapes were congruent, by equating equal sides and angles for the pairs of shapes. This activity gave learners practice with the conventions of recording statements when solving problems.

The worksheet also included some similar shapes. The lesson concluded with a discussion on shapes that *looked* the same, but did not fulfill all the requirements of congruency.

Day 4
The focus shifted to similarity. There was limited time for learners to engage with the concept enlargement and reduction, so I showed them how to produce an enlarged or reduced polygon from the original. In our class discussion we concluded that in similar shapes the corresponding angles are equal, but the sides are not. Learners were then handed a worksheet where they had to measure sides of similar shapes. They were led to the conclusion that the sides of similar shapes are in proportion.

Day 5
This lesson was spent on consolidation of the properties of congruency and similarity, and the difference between the two. Learners were handed a worksheet where they had to identify whether sets of triangles and quadrilaterals were congruent or similar. For the first time they worked with sketches that were not drawn to scale, and had to rely on the markings and measurements on the shapes. This is an example:



Learners had to write sets of equations to conclude that $\Delta ABC \equiv \Delta RPQ$

CONCLUSION

This methodology of integrating various strands, takes careful planning. The oversights in my first lesson forced me to think very hard about how I would integrate the other content strands effectively. In the end, learners responded well to the set of lessons and I am confident that they understand the properties of, and difference between similarity and congruence. The next five lessons were devoted to using the properties of similarity and congruence to solve problems.

An advantage of integrating different topics when teaching a particular topic, is that learners grow to accept that no mathematical concept stands in isolation. In Term 3 when we cover Transformation Geometry, the properties of Similarity and Congruency will be revisited.

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HOW I TEACH FRACTIONS IN THE INTERMEDIATE PHASE WHEN THE LANGUAGE OF LEARNING AND TEACHING (LOLT) IS ENGLISH AND THE LEARNERS' HOME LANGUAGE IS NOT ENGLISH

Nombeko Mafenuka, Janine Wilson

Pearson South Africa

INTRODUCTION

In 2004, Myburgh *et al.* reported that where learners do not speak the language of instruction, authentic teaching and learning cannot take place. The Department of Basic Education agrees and states that such a situation largely accounts for the school ineffectiveness and low academic achievement experienced by students in Africa (DBE, 2010).

We feel that since the LOLT in Intermediate Phase Mathematics is English in many schools in South Africa – 80% of Intermediate Phase Mathematics classes were taught in English in 2007 (DBE, 2010) – it is important that teachers have the necessary tools to support and assist their learners in this crucial phase.

Therefore, we believe that introducing English terminology about fractions in Grade 4 should be done slowly, with plenty of repetition and with the use as many interesting visual resources as possible.

SIX THINGS TO CONSIDER BEFORE YOU START A LESSON

1. Make sure you explain to learners why it is important to learn about fractions. Use pictures or props wherever possible. For example:

- Fractions are important because we encounter them in our everyday lives:
    - Fractions are used in time, such as quarter past, half past, half a day, quarter of a year. Show them a clock face to illustrate some of these times.
    - When cooking and baking we measure ingredients using words like: half a dozen eggs, half of a tablespoon, a quarter of a cup.
    - In shops we often see signs that say ‘Half price sale!’

2. Reinforce vocabulary in order to boost their language development. For example, go through these words one at a time explaining each one and illustrating each word using pictures or drawings wherever possible:

whole number, half, double, quarter, third, tenth, divide, share, equal parts, denominator, numerator, common fraction, proper fraction, mixed fraction, equivalent fraction

3. Use visual resources that will make it easier for the learners to understand the concepts you are trying to explain. For example:

- laminated fraction strips
- different coloured shapes cut into fractions and labelled
- fraction walls and number lines
- flash cards with fraction notation and words
- pictures of everyday objects that can be divided into fractions.

4. Explicitly teach thinking skills rather than have learners rote learn the subject. Remember that learners should be exposed to different types of questions that will make them think creatively, even if they haven't yet mastered the language. For example:

- '54 ÷ 6 = 9' **or** '54 sweets shared between 6 children'.
- '54 ÷ 2 = 27' **or** 'Halve 54' **or** 'What is half of 54?'
- Ask learners: 'Why are $\frac{1}{2}$ and $\frac{5}{10}$ the same?' Instead of asking them 'Which fraction is equivalent to $\frac{5}{10}$?'

5. Construct a multiple meaning bank that will help learners know a question whichever way it is been posed. For example:

- 'What is one fourth of twelve?' **or** 'What is a quarter of twelve?'

6. Use collaborative learning strategies to help learners to learn from each other. For example:

- The way your learners sit in class should be determined by the kind of activity you will be doing. If you have been teaching a lesson that is difficult for some learners to grasp, mix those who struggle with those who have understood the lesson well and let them help each other.

THE LESSON

Lesson objectives

Note that you may not get through all of these objectives in one lesson. Be aware of whether learners are keeping up and teach at the correct pace for your learners.

At the end of the lesson learners will be able to:

- understand prerequisite knowledge using new terminology in English, i.e. sharing and fractions
- define fraction, numerator and denominator (in English)
- identify the number of shaded parts and the number of equal parts in a shape

- recognise and write a fraction using mathematical notation and using words
- determine if two fractions are equivalent using shapes
- add fractions with the same denominators.

Starting off

Show learners a bar of chocolate (or any other physical resource that can be divided in different ways) and ask them: 'If I have one bar of chocolate and I want to give it to two children, what should I do to make sure that each child gets an equal share?' Illustrate this using two learners in the class, or using pictures or drawings.

Then discuss: 'If I want to give the same bar of chocolate to three children, how will I make sure that each child gets an equal share?'

And lastly: 'If I want to give the same bar of chocolate to four children, how will I make sure that each one gets an equal share?'

Write down answers as they give them to you.

Once you've written all the answers down, ask the learners how they worked out their answers and discuss them together. If learners do not easily come up with the correct answers, make representations of the chocolate using cardboard or paper, cut the drawing up into pieces and share the pieces between the children.

Activities

1. Divide your learners into groups and give each group one bar of chocolate (or any other physical resource that can be divided in different ways). Make sure the chocolate can be divided equally amongst the number of children in the group but try to have groups of different sizes (for example, if you have bars of chocolate that have 12 pieces, divide the learners into groups of 2, 3, 4 and 6). See how they share the chocolate you have given them. Make sure that each child in a group gets an equal share of the group's chocolate.

   Then ask them to look at the pieces they have shared and talk in their groups about what fraction each member of the group has. Then ask different groups to compare how the chocolates have been divided differently. Discuss who got the biggest portion and why. (This will be determined by the number of learners in the group – if there were many learners in a group, each learner will get a smaller fraction of the chocolate.)

   Ask one learner from each group to come to the front of the class with their pieces and then ask the rest of the learners to arrange them from most to least (biggest fraction of the whole to smallest fraction of the whole), according to the pieces they have in their hands. Let them discuss which fraction is bigger and which fraction is smaller.

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2. Write a fraction on the board, or hold up a fraction card for everyone to see and ask them if they know what this is called. (For example: $\frac{1}{2}$)

Expected answer: Fraction

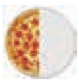
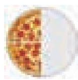
Then ask them to explain what a fraction is.

Expected answer: A fraction is a part of a whole.

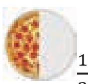
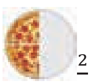
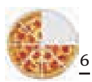
Tell learners that in fraction we have two numbers. Ask them if anyone can tell you what these numbers are called.

Expected answer: The top number is called the numerator and the bottom number is called the denominator.

3. Show learners some shapes with shaded parts and ask them to tell you which fractions they represent. Make sure that learners can match the fractions in fraction notation and also in words. For example:

					
$\frac{1}{2}$ one half	$\frac{1}{4}$ one quarter	$\frac{3}{8}$ three eighths	$\frac{2}{4}$ two quarters	$\frac{4}{8}$ four eighths	$\frac{3}{4}$ three quarters

Ask your learners to look at the fractions below and say which ones they think are different but have the same value. Tell them that these fractions are called equivalent fractions. Give them a chance to choose those fractions that are equivalent to each other. For example:

 $\frac{1}{2}$	 $\frac{1}{4}$	 $\frac{3}{4}$
 $\frac{2}{4}$	 $\frac{2}{8}$	 $\frac{6}{8}$

4. Ask your learners if they know how to add fractions together. Go over the difference between the numerator and the denominator of a fraction again. Then explain that they can only add fractions together when the denominators are the same. Show them physical examples or drawings so that they understand why this is the case. Then show them this example:

$$\frac{5}{10} + \frac{3}{10} = ?$$

- The denominators are the same, so we can add the numerators.
- Add the top numbers and put the answer over the denominator.

  $$= \frac{5+3}{10} = \frac{8}{10}$$

  Illustrate this by counting fraction pieces, colouring in drawings as you count, or 'hopping' in eighths on a number line. Do not expect learners to grasp the notation straight away. Use many examples to show them how to do this.

ADDITIONAL NOTES AND TIPS

- Explain terms in English using simple language wherever possible.
- Use as many visual resources as you can, including physical resources that learners can manipulate using their hands.
- Use a class test to find out what learners know and what they don't know, and immediately intervene as soon as you discover problems.
- Talk to the parents of learners who are struggling. The best is to call a group of parents in and show them how to help their children at home. If they are unable to help, advise them to ask trustworthy friends, family members or even an older learner to help their child after school.
- Encourage learners to talk about how they got to their answers so that you can easily find mistakes and misunderstandings they may have.
- Be sensitive and take extra care considering those learners who find fractions difficult and frustrating. Always be patient with these learners. It is much better and easier for you if learners love the subject rather than hate it.
- Praise learners for each little improvement they show and always tell them how great they can be if they practise working with fractions everyday.
- Many learners and teachers think of fractions as the most difficult area to understand in Mathematics. Try to keep your lesson positive and make sure that learners believe that they **can** do it. Always try to make learning fractions fun and easy.
- Try to bring sweets, chocolates, pizzas or biscuits to work with in class, and let learners share them at the end of the lesson.

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Department of Basic Education. 2010. The status of the language of learning and teaching (LOLT) in South African public schools.

Myburgh, O., Poggenpoel, M. and Van Rensburg, W. 2004. Learners' experience of teaching and learning in a second or third language. Education, 124(3):573.

TITLE: HOW I TEACH PATTERNS IN A GRADE ONE CLASS

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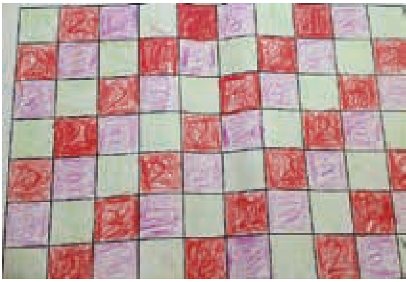
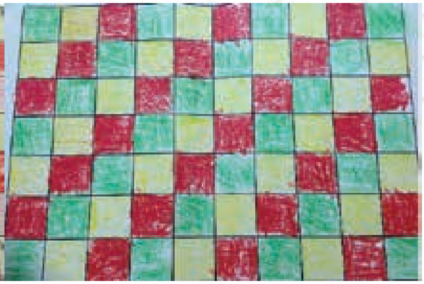
INTRODUCTION

Patterns are things, numbers, shapes, images that repeat in a logical way. Pattern help children learn to make predictions, to understand what comes next, to make logical connections, and to use reasoning skills. The elements of a pattern repeat in a predictable manner. Patterns can be based on a template or model which generates pattern elements, especially if the elements have enough in common for the underlying pattern to be inferred, in which case the things are said to exhibit the unique pattern www.wikipedia.org. Finding and understanding patterns gives us great power. With pattern we can learn to predict the future, discover new things and better understanding of the world around us www.mathisfun.com. Numbers can be arranged into a pattern, you can make your own patterns with numbers. Number patterns such as 1; 2; 3; …….. are familiar to Foundation Phase learners, these are first patterns they learn. As learners advance they learn number patterns with different sequences. Patterns can also be formed with shapes, pictures, colours and words.

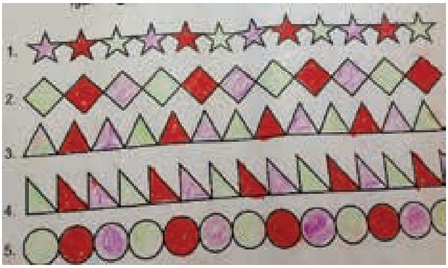
PRESENTATION

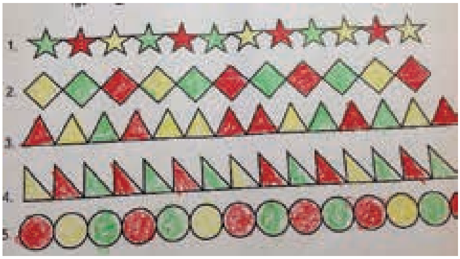
In the first demonstration lesson learners were given square grid papers to write numbers 1; 2; 3 in the first three squares. They were to repeat these numbers until they filled in the whole grid. As they were sitting in groups of three in a desk, they were expected to turns in writing these number on the grid. After completing the writing of numbers they were asked to choose a colour for each number and then colour each square with the selected number, for example if they choose green for number 1, red for 2 and pink for 3 like in the example on the picture below. The competent groups were later given a smaller grid to complete as an individual activity.



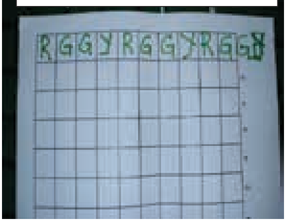


In the second demonstration lesson they were given different shapes to colour using their own design. They were also asked to take turns in colouring the design. Others managed to repeat a specific pattern some just colour the shapes to make beautiful decoration but failing to produce a repeating pattern out of the chosen colours.





The third lesson was on application where learners were expected to first make a design of the colours of unifix cubes they will use to make a necklace. A demonstration on how to make a design on the grid was first conducted together with the class. One group was asked to do an activity for the class by first selecting the colour of the unifix required for the design. A demonstration on how to thread the unifix following the pattern on the grid was shown to the learners.



Demonstration Grid for necklace design.

The groups were then asked to make their own designs with not more than three different colours. After finishing the design they collected the colours of the unifix they need for their necklace.

The majority managed to create good patterns but others were still struggling in identifying the pattern they have followed in their design. The picture below

shows the products of different groups as I have stated that others did not managed to follow specific pattern in their design. What I have observed in these lessons is that learners enjoyed working with concrete objects in making their patterns.

The last demonstration lesson was a class activity which combines colour and shapes. We were designing a T-shirt changing it from a plain white to a colourful garment. A plain white T-shirt was pasted on the board, learners were asked to select the shape they will use to design the shirt. They first selected a triangle and a row of triangles with alternating colours was pasted by learners taking turns on the shirt. Different shapes and colours were used and the picture below was a complete product of learners' design with shapes and colours.

Demonstration Grid for necklace design

Conclusion

In our discussion with the teachers who observed the lesson; they highlighted high learner participation in the lessons. The majority of learners were able to finish their individual activity after completing the group activity. The teachers were able to see that the group activity provided learners with opportunity of peer support, learners learn more from others while doing an activity in a group. When asking learners to explain how they created their patterns assisted the teachers with ideas of engaging learners not only with the "what" questions but also the "how". Learners were also able to predict if you give them an idea of the pattern and also count the number of unifix they needed to complete their necklace.

REFERENCES

1. www.wikipedia.org
2. www.mathisfun.com

TEST AND IMPROVE
PAPER-BASED AND ONLINE TESTING SOLUTION

Presenting Author: Tatiana Sango

Pearson

The Poster Presentation will provide a visual explanation of Test and Improve, a targeted and effective diagnostic assessment programme for Grades 8 to 12 based on the Grade 12 matric exams. The presentation will detail the process of how the paper-based version, as well as the online version, is designed to test curriculum knowledge and skills, diagnose learners' development areas and identify the sub-skills requiring intervention, leading to improved results in Maths and Physical Sciences.

POSTER 1

Test and Improve: An effective process

The poster will visually explain the four steps needed to test learners' skills and knowledge:

- Learners write paper-based tests
- Tests are processed using psychometric software
- Rich diagnostic reports are generated
- Reports help understand learners' development areas

POSTER 2

How it works: Test learners' skills and knowledge

Grade 12 exam questions are deconstructed to identify learners' skills that are problematic by testing the sub-skills the learner should have acquired from previous grades.

- A visual detailing how the Grade 8-11 sub-skills are broken up to test the Grade 12 exam question.

POSTER 3

How it works: Rich diagnostic reports to help learners' flourish

Reports provide teachers with learner level, class level and grade level results. The reports are granular and comprehensive, and give a lot of detail about the skills that need attention, allowing teachers to design targeted interventions and planning.

Summary skills are ranked according to performance, from the skill with the lowest score to the skill with the highest score. This enables teachers to target the skills with the lowest scores for intervention.

- A visual illustrating the graphic representation of results
- A visual illustrating the types of reports received

POSTER 4

Advantages of Test and Improve:

Test and Improve has been piloted with more than two thousand learners across South Africa. Teachers have commented extensively on the value of this programme in a variety of focus groups. One teacher commented that, "this is an extremely valuable report to provide real scientific data and pinpoint exactly what the issue is and therefore provides a chance of rectifying the problem".

There are several different purposes that Test and Improve is used for:

- Diagnose learners' skills and knowledge development areas.
- Identify sub-skills which require the most attention and development.
- Grade 8 testing to use as a baseline diagnostic.
- Grade 9 testing to inform subject choices for Grade 10.
- Grade 10 and 11 testing to ensure a firm knowledge and skills base for Grade 12.
- Early Grade 12 testing to improve learners' Grade 12 results.

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POSTER 5

Test and improve online:

We will have a look at the online version of Test and Improve which will enable parents and learners to take the test online and independently assess their skills and knowledge.
The CD-based Baseline Assessment Tool will offer the following features and benefits:

- Quick return of learners' results and diagnostics.
- Increased time for the teachers to prepare and deliver targeted and individualised intervention strategies based on the identified content and skills gaps.
- Reduced administrative burden on the teachers, freeing them from the test development, marking and results analysis tasks at the beginning of the year.
- An interactive, non-threatening and engaging way for the learners to complete the baseline assessment and as a result display their knowledge and skills.
- The ability to include 'technology-enhanced' testing items that allow learners to engage with the test in ways other than multiple choice questions that also may demonstrate critical-thinking skills.

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HOW I TEACH NUMBER PATTERNS

Lloyd Stuurman

Strandfontein High School, Western Cape

This 'How I Teach' presentation demonstrates how two consecutive topics from the Grade 8 CAPS schedule for Grade 8, namely Number and Geometric Patterns *and* Functions and Relations *were compressed into one set of seven lessons.*

INTRODUCTION

One of the challenges of high school mathematics is getting to grips with the abstract concepts presented in Patterns, Functions and Algebra. This is the first year that I experience having to pace my teaching according to CAPS. Only a week is allowed for patterns. Using an extra day for patterns will result in falling behind with the required pace that I should be working at. Bear in mind that the same is true with every other topic in mathematics. When preparing for the set of lessons, I considered this quote from the Senior Phase CAPS document:

In Patterns, Functions and Algebra, learners' conceptual development progresses from … the recognition of patterns and relationships, to the recognition of functions, where functions have unique output values for specified input values.

(Senior Phase Mathematics Curriculum and Assessment Policy Statement, 2011, page 21)

Rationale

I realized that number patterns provide an opportunity to lead learners to the point where they recognise functions based on the relationship between the input and output values. Topic 2.2, *Functions and Relationships* states that learners should "determine input or output values or rules or patterns and relationships using flow diagrams, tables, formula, equations" (2011: 22). In Term 1 the topics *Numeric and Geometric Patterns* and *Functions and Relationships* follow one another. There are 4,5 hours and 3 hours allocated to the two topics respectively. I decided to merge the content with the aim of helping learners to recognise functions where functions have unique output values for specified input values.

Within the topic *Numeric and Geometric Patterns* Grade 7 and 8 learners should investigate and extend numeric and geometric patterns and represent their solutions in tables. By Grade 8 they are expected to also represent the rule algebraically and the range of number patterns is extended to include patterns with multiplication and division of integers and numbers in exponential form. This provides an opportunity for learners to engage with basic number concepts.

I also consider it important to make the distinction between 'position of the term' and 'term' in a sequence. This prepares the way for introducing the terminology 'input-' and 'output-value'. Investigating number patterns is an opportunity to generalize, to give general algebraic descriptions of the relationship between terms and their position in a sequence and to justify solutions.

For the topic Functions and Relationships, learners are expected to determine input and output values or rules for patterns and relationships using flow diagrams, tables, formulae and equations, with the latter being the only extension from Grade 7 to 8. They are also expected to recognise the equivalence between the different representations.

CONTENT

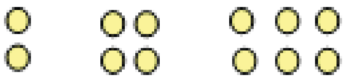
By combining the two topics I afford learners 7,5 hours to understand these two related concepts. This amounts to approximately 7 lessons. The content of the lessons is outlined below.

Day 1: Investigating and Extending Patterns
I started with a few revision exercises just to refresh the memory of the learners. The exercise below is an example of work covered.

Consider the following sequence:

4 ; 7 ; 10 ; 13

or

Consider the pattern 

Give the difference between the consecutive terms.

Describe the rule you found in words.

What would the values of the next 3 terms be?

What would the value of the 10th term be?

As mentioned earlier, for the learners to understand these questions, they would have to know the definition of rule and term. It is therefore important to revise these terms before giving these exercises. The first day could basically be spent on identifying patterns with common differences and ratios, and describing the relationship between terms and the term number. This should be a familiar task as learners covered this ground in Grade 7.

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Day 2: Getting learners to

- **extend geometric and numeric patterns**
- **complete the table representing the pattern, by filling in the missing output values**

Learners would be expected to extend a pattern in the table, where some terms are missing. They would be expected to find the general rule. This is also a revision task, but many learners may struggle.

Day 3

Same as Day 2, but including a flow diagram where learners will fill in the operations determined by the general rule.

This task takes learners to a level where, besides determining the general rule, they are expected to break up the rule into its different operations in their correct order. This forces them to consider the order of operations, an important inclusion in Topic 1: Number, Operations and Relations. I include excerpts from a Day 3 task:

Learners were given a pattern like the one below:

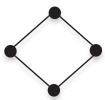


FIGURE 1

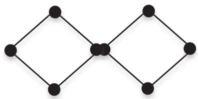


FIGURE 2

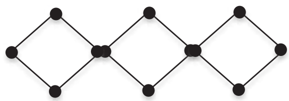
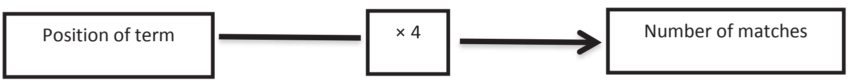


FIGURE 3

After doing the basic tasks of extending the pattern and completing a simple table by filling in missing output values, most were able to see that the rule was "multiply the term number by 4". They were then given this flow diagram:



The value of this flow diagram is that it highlights the relationship between the input value (position of the term) and the output value (term i.e. number of matches). Learners would be challenged to complete a table based on the rule represented in the flow diagram.

Position of term (n)	8	12	15	22	31	50
Number of matches (m)						

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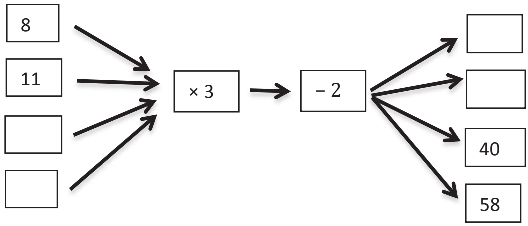
Day 4

Learners continue to work with flow diagrams, most representing rules comprising multiple operations. For example, the geometric pattern that yields the rule $m = 3n - 2$ is presented to learners, who are expected to fill in the missing outputs and input:



To derive the missing input, learners will realize that they need to apply inverse operations to those indicated in the flow diagram. This is a powerful way to introduce inverse operations.

They would also be expected to write the rule as an algebraic equation, and to complete a table.

Days 5, 6 & 7

Learners consolidate their understanding of number patterns and are expected to apply inverse operations to derive input values from given outputs.. By Day 6 I introduce the notion of constant difference and constant ratio, and look at functions where there is neither constant difference nor ratio. I show them a systematic method to derive the rule for a number pattern.

CONCLUSION

Using this strategy has many advantages. In the past, learners really struggled to find the *n*th term. This method underscores the notion of functional relationships between the input and output value and helps them to derive the rule, which is generalized as the equation containing the *n*th term. When moving on to algebraic equations later in the year, it will also aid learners to understand the concept of applying the inverse operations to solve for x and using the "check" method to prove that the answer is true.

REFERENCES

Department of Basic Education, 2011. National Curriculum Statement; Curriculum and Assessment Policy Statement., Mathematics, Grade 7 – 9. Department of Basic Education, Pretoria.

Lewis, A. et al, 2012. Mathematics Companion Learner's Workbook Grade 8 Volume 2. The Answer Series, Claremont.

HOW I TEACH
"SOLVING 3-DIMENSIONAL PROBLEMS IN TRIGONOMETRY"

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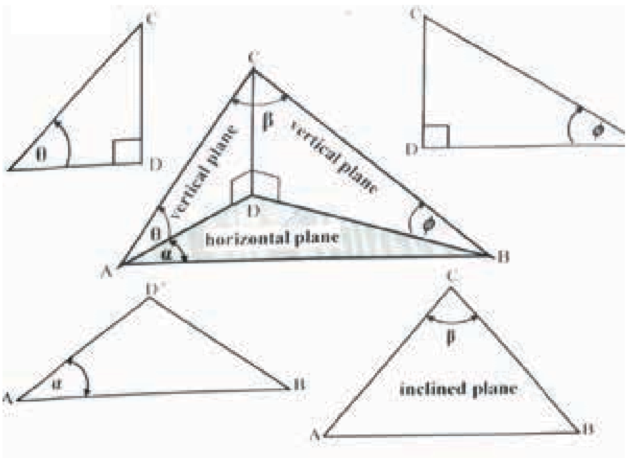
INTRODUCTION

Three dimensional problems are usually solved by taking a succession of triangles in different planes and applying to each separately the results which are already established (Area, sine and cosine rules).

But in cases of right angle triangles definitions of sine $\left(\frac{O}{H}\right)$; cosine $\left(\frac{A}{H}\right)$; tangent $\left(\frac{O}{A}\right)$ are used. It is always necessary and sufficient to start with the triangle with the most information (like 2 sides and an angle, etc.)

SOLUTION OF TRIANGLES IN THREE DIMENSIONAL DIMENSIONS:

Whereas two-dimensional space occupies a single plane, three-dimensional space occupies three planes. The three planes are horizontal, vertical and inclined. The sine, cosine and area rules can also be used to solve problems in three dimensional space. The diagram below illustrates the three different planes for an object in three dimensions.

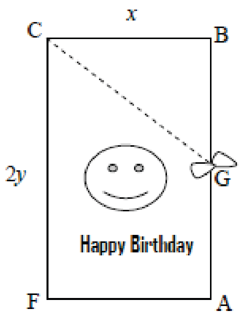


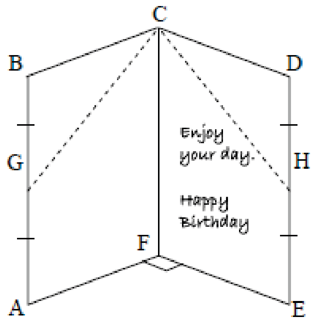
WORKED EXAMPLES

Question 1: March / Feb 2011

A rectangular birthday card is tied with a ribbon at the midpoints, G and H, of the longer sides. The card is opened to read the message inside and then placed on a table in such a way that the angle $A\hat{F}E$ between the front cover and the back cover of the card is $90°$. The points G and H are joined by straight lines to the point C inside the card, as shown in the sketch.

Let the shorter side of the card, BC = x, and the longer side, CF = $2y$.

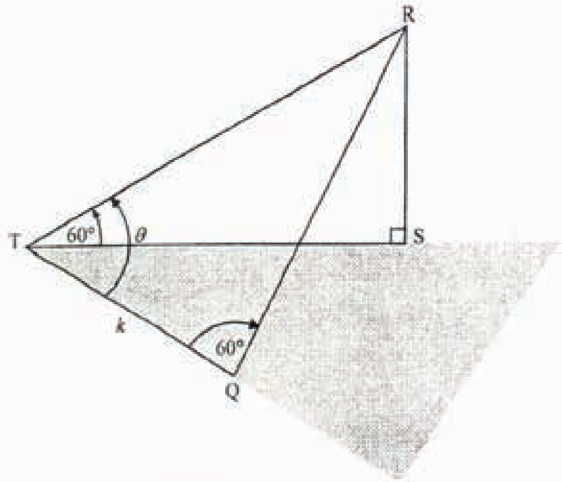




Prove that $\cos G\hat{C}H = \dfrac{y^2}{x^2 + y^2}$. **[8]**

Question 2: March/ Feb 2014

In the diagram below, RS is the height of a vertical tower. T and Q are two points in the same horizontal plane as the foot S of the tower. From point T the angle of elevation to the top of the tower is 60°. $R\hat{T}Q = \theta$, $R\hat{Q}T = 60°$ and TQ = k metres.

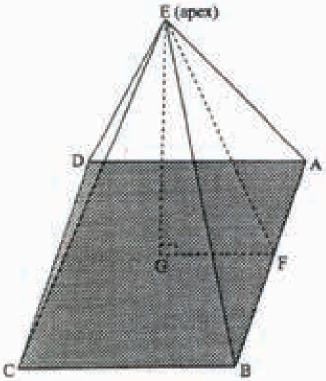


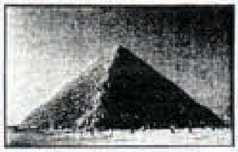
10.1 Express TR in terms of θ and k. (3)

10.2 Show that $RS = \dfrac{3k}{2(\sqrt{3}\cos\theta + \sin\theta)}$. (7)

Question 3: Dec 2013

The Great Pyramid at Giza in Egypt was built around 2 500 BC. The pyramid has a square base (ABCD) with sides 232,6 metres long. The distance from each corner of the base to the apex (E) was originally 221,2 metres.





Great Pyramid at Giza in Egypt

13.1 Calculate the size of the angle at the apex of a face of the pyramid (for example $C\hat{E}B$). (3)

13.2 Calculate the angle each face makes with the base (for example $E\hat{F}G$, where $EF \perp AB$ in ΔAEB). (6)

[9]

TRIAL EXERCISES

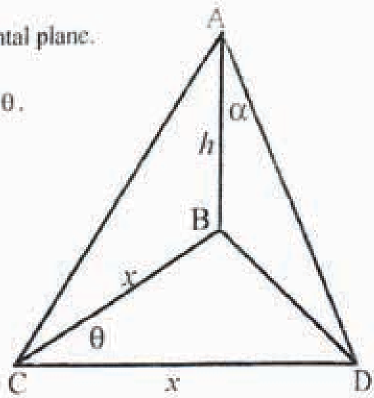
EXERCISE 1

In the following sketch, ΔBCD is in a horizontal plane.
A is directly above B (AB is a vertical line).
$AB = h$, $BC = CD = x$, $B\hat{A}D = \alpha$ and $B\hat{C}D = \theta$.

(a) Show that $BD^2 = 2x^2(1-\cos\theta)$

(b) Hence show that $h = \dfrac{x\sqrt{2(1-\cos\theta)}}{\tan\alpha}$

(c) Calculate the value of h if
$x = 100$, $\theta = 60^\circ$ and $\alpha = 40^\circ$
(two decimal places)



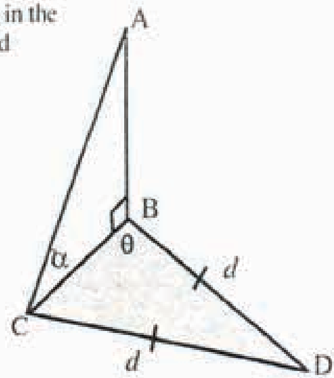
EXERCISE 2

Refer to the figure. B, C and D are three points in the same horizontal plane so that $BD = CD = d$ and $C\hat{B}D = \theta$. AB is perpendicular to the plane. From C the angle of elevation of A is α.

(a) Express $\hat{D}$ in terms of θ.

(b) Prove that: $AB = 2d\cos\theta\tan\alpha$

(c) If $d = \sqrt{2}$ units; $\alpha = 30°$ and $\theta = 75°$ calculate AB without the use of a calculator.



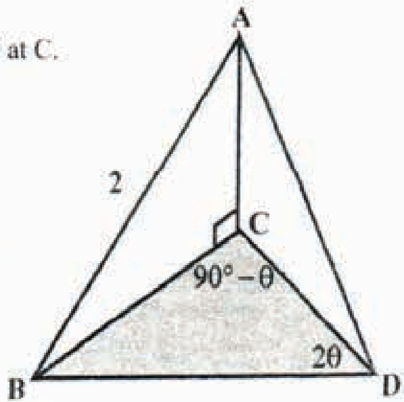
EXERCISE3

AC represents a vertical tower which is perpendicular to the horizontal plane BCD at C. AB is 2 units. $B\hat{C}D = 90° - \theta$, $B\hat{D}C = 2\theta$ and $B\hat{A}C = \theta$.

(a) Determine BC in terms of θ.

(b) Show that BD = 1 unit.

(c) If $AD = \sqrt{3}$ units, calculate the size of $A\hat{B}D$ without using a calculator.



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NOTES